المستوى الثالث

احصاء

1969	Tanta University Faculty of Science Department of Mathematics		
	Final term exam for the Second semester 2016-2017		
	Course title:	Optimal Control +Functional Analysis	Course code: MA3210
	Date:4 /6/2017	Total Marks: 150	Time allowed: 2 Hours

Answer all the following questions:

First question: (45 Marks)

(a) Show that the set $S=\{(x_1,x_2)\in \mathrm{R}^2 : |x_1|+|x_2|<1\}$ is convex set.

(b) Find a necessary condition for a function to be an extremal for the functional $J(x)=\int_{t_0}^{t_f} F(x(t),x'(t),t)\,dt$, where $t_0, x(t_0), t_f\ x(t_f)$ are specified?.

(c) Show that:

(i) Euler's equation can be written in the form: $\frac{d}{dt}\left(F-x'\frac{\partial F}{\partial x'}\right)-\frac{\partial F}{\partial t}=0$.

(ii) If F is not an explicit of t, then $F-x'\frac{\partial F}{\partial x'}=c$.

Second question: (30 Marks)

(a) On what curves can the functional:

$$J[x(t)]=\int_1^2 \left(x'^2-2xt\right)dt,\quad x(1)=0,\quad x(2)=-1$$

attain an extremum.

(b) Optimize $J(x)=\int_{t_0}^{t_f}\left(2x'^2-42xt+11t\right)dt$ s.t. $x(t_0)=x_0$ and $x(t_f)=x_f$

(c) Find the extremals of the problem corresponding to the functional

$$J=\int_0^{\frac{\pi}{2}}\left(2x_1x_2-2x_1^2+x_1'^2-x_2'^2\right)dt \text{ s.t. } x_1(0)=0, x_1\left(\frac{\pi}{2}\right)=1, x_2(0)=0, x_2\left(\frac{\pi}{2}\right)=1.$$

P.T.O

Third Equestion: **(40 Marks)**

(a) Define the following:

(1) Bounded set in metric space (X,d).

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(4) Complete metric space (X,d). (5) Incomplete metric space (X,d).

(6) Contraction mapping (7) Continuous mapping. (8) Linear space.

(9) Normed space $(X,\|\cdot\|)$ (10) Fixed point

(b) State and prove Cauchy's inequality. Hence or otherwise prove that the space (R^n,d) is complete metric space while the metric space $((0,1],d)$ is incomplete metric space.

(c) Prove that the space l_p with $\|x\|=\left(\sum_{j=1}^{\infty}|x_j|^p\right)^{\frac{1}{p}}$ is Banach space.

Fourth Equestion: **(38 Marks)**

(a) If $f(x)$ and $\varphi(x)\in L_2(a,b)$, moreover $\int_a^b\int_a^b k^2(x,y)\,dx\,dy<\infty$. Prove that the Fredholm integral equation $f(x)=\varphi(x)+\lambda\int_a^b k(x,y;f(y))\,dy$ has a unique point if $|\lambda|<\left(\int_a^b\int_a^b k^2(x,y)\,dx\,dy\right)^{-\frac{1}{2}}$.

(b) Prove that the functions " $d(x,y)$, contraction mapping and $\|x\|$ " are continuous functions.

(c) If $S=\{x: x=(\xi_1,\xi_2,\cdots,\xi_n,\cdots),\xi_i\in R\ \forall j=1,2....\}$ and $d(x,y)=\sum_{j=1}^{\infty}\frac{1}{2^j}\frac{|\xi_j-\eta_j|}{1+|\xi_j-\eta_j|}$.

Prove that:

(1) (S,d) is metric space.

(2) $(S,\|\cdot\|)$ is not normed space, where $\|x\|=d(x,0)$.

(d) In R^n, we define $d_1(x,y)=\max\{|x_j-y_j|\}$, $d_2(x,y)=\sum_{j=1}^{n}|x_j-y_j|$ and $d_3(x,y)=\sqrt{\sum_{j=1}^{n}(x_j-y_j)^2}$. Prove that $d_1,\ d_2,d_3$ are equivalent.

Examiners:	1- Prof. Dr. S. Abdel Aziz-	2- Dr. N. El-Kholy

١٠ امتحان

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TANTA UNIVERSITY
FACULTY OF SCIENCE
DEPARTMENT OF MATHEMATICS

EXAMINATION FOR SOPHOMORES (THIRD YEAR) STUDENTS OF STATISTICS				
COURSE TITLE:	Statistical Programs			COURSE CODE: ST 3204
DATE: 28/5/2017	TERM: SECOND	TOTAL ASSESSMENT MARKS: 150		TIME ALLOWED: 2 HOURS

Final Examination Paper

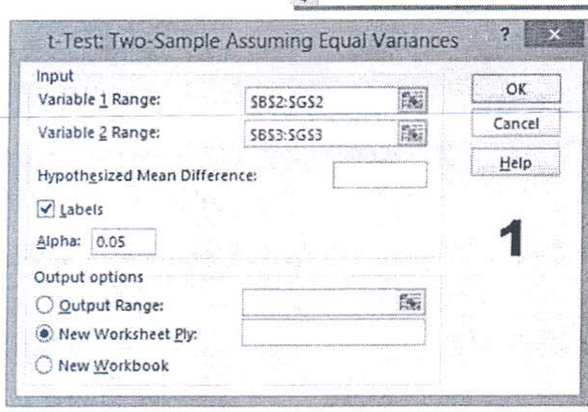
Answer the following questions:

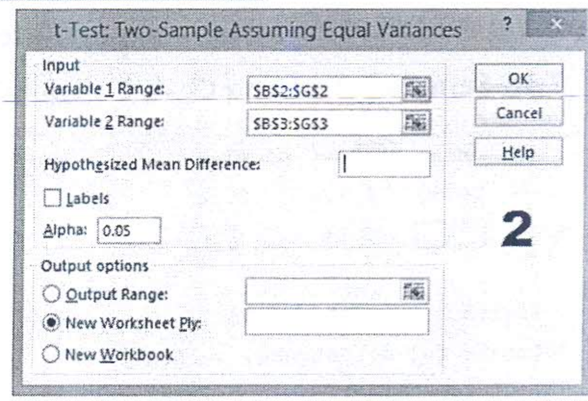
Q1: a. Construct a questionnaire consists of at least 10 questions about any topic (Arabic language can be used).

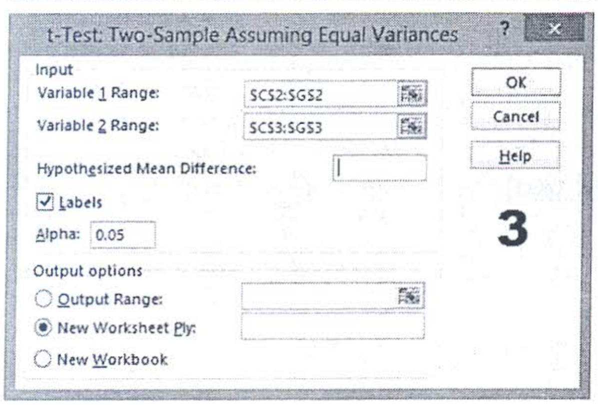
b. Show that the two-sample t-test (unequal variance) can be used in excel to perform the one-sample t-test.

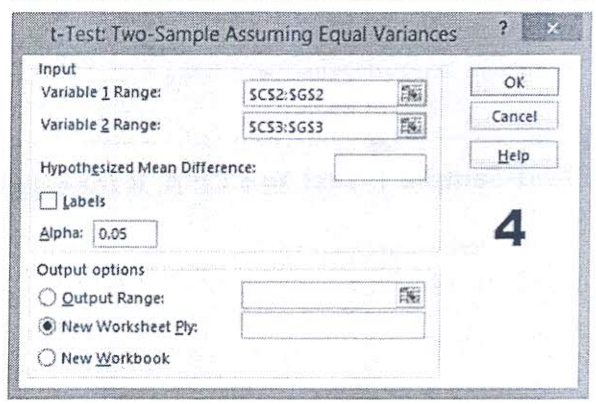
c. Consider the given data in Excel; explain in details the difference between the following four pictures then write down the result in each case:

	A	B	C	D	E	F	G	H
1								
2		A	23	15	43	19	26	
3		B	26	18	26	31	14	
4								









EXAMINERS	DR. ELSAEED AMMAR	DR. MOHAMED M. EZZAT

Q2: a. From the following table, write the regression equation:

	Coefficients	Standard Error	t Stat	P-value
Intercept	19.188	4.551	4.216	0.003
X	1.016	0.294	3.457	0.010

b. Using this table, complete the following sentence:

Source of variation	SS	df	MS	F	P-value
Blocks	27.33	3	9.11	....	0.0126
Treatments	1038.50	2	519.25	....	0.0002
Error	6.17	6	1.02		
Total	1072	11			

The name of this test is ....; the number of blocks is .... while the number of treatments is .... The values of the two F statistics are .... and .... Using the p-values we can say that .... between treatments and .... between blocks.

Q3: Complete the following tables and run the test:

a. **Two-Sample T-Test and CI: A, B (<u>two-tailed test</u>)**

```
   N   Mean   StDev  SE Mean
A  6  16.50    1.87     0.76
B  9  [    ]   3.16     [  ]

Difference = μ (A) - μ (B)
Estimate for difference:  -16.50
95% CI for difference:  (-19.62, -13.38)
T-Test of difference = 0 (vs [ ]): T-Value = [    ]   P-Value = 0.000   DF = [ ]
Both use Pooled StDev = 2.7386
```

b. **Two-Sample T-Test and CI: A, B (<u>one-tailed test</u>)**

```
   N   Mean   StDev  SE Mean
A  9  15.89    1.90     0.63
B  7  [    ]   2.37     [  ]

Difference = μ (A) - μ (B)
Estimate for difference:  -16.68
95% upper bound for difference:  -14.71
T-Test of difference = 0 (vs [ ]): T-Value = [    ]   P-Value = 0.000   DF = 11
```

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c. Paired T-Test and CI: D, E (two-tailed test)

	N	Mean	StDev	SE Mean
D	8	24.75	[ ]	1.10
E	8	45.00	2.83	[ ]
Difference	8	[ ]	1.982	0.701

95% CI for mean difference: ([ ], [ ])

T-Test of mean difference = 0 (vs [ ]): T-Value = [ ] P-Value = 0.000

d. One-Sample T: C (one-tailed test)

Test of μ = 20 vs [ ]

Variable	N	Mean	StDev	SE Mean	95% Upper Bound	T	P
C	12	16.250	2.340	[ ]	[ ]	[ ]	0.000

e. One-way ANOVA: A, C, E

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Factor	[ ]	[ ]	[ ]	391.36	0.000
Error	[ ]	133.7	[ ]		
Total	25	[ ]			

Critical Values of t-Distribution

df	0.1000	0.0500	0.0250	0.0100	0.0050	0.0010	0.0005
1	3.078	6.314	12.706	31.821	63.657	318.309	636.619
2	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850

Good Luck

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