Solid State Physics II

Level 4

Semester 1

Course Content

L1. Introduction to solid state physics - The free electron theory : Free levels in one dimension.

L2. Free electron gas in three dimensions.

L3. Electrical conductivity – Motion in magnetic field- Wiedemann-Franz law.

L4. Nearly free electron model - origin of the energy band.

L5. Bloch functions - Kronig Penney model.

L6. Dielectrics I : Polarization in dielectrics

L7 .Dielectrics II: Types of polarization - dielectric constant

L8. Assessment

L9. Experimental determination of dielectric constant

L10. Ferroelectrics (1) : Ferroelectric crystals

L11. Ferroelectrics (2): Piezoelectricity

L12. Piezoelectricity Applications

L1 : Solid State Physics

- Solid state physics is the study of rigid matter, or solids, ,through methods such as quantum mechanics, crystallography, electromagnetism and metallurgy.
- It is the largest branch of condensed matter physics.
- Solid-state physics studies how the large-scale properties of solid materials result from their atomic-scale properties.
- Thus, solid-state physics forms the theoretical basis of materials science.
- It also has direct applications, for example in the technology of transistors and semiconductors.

Crystalline solids & Amorphous solids

- Solid materials are formed from densely-packed atoms, which interact intensely.
- These interactions produce :

  the mechanical (e.g. hardness and elasticity),

  thermal,

  electrical,

  magnetic

  and optical properties of solids.

- Depending on the material involved and the conditions in which it was formed

, the atoms may be arranged in a regular, geometric pattern (crystalline solids, which include metals and ordinary water ice)

, or irregularly (an amorphous solid such as common window glass).

Crystalline solids & Amorphous solids

- The bulk of solid-state physics theory and research is focused on crystals.

- Primarily, this is because the periodicity of atoms in a crystal facilitates mathematical modeling.

- Also, crystalline materials often have electrical, magnetic, optical, or mechanical properties that can be exploited for engineering purposes.

Free Electron Theory

- Properties of materials such as electrical conduction and heat capacity are investigated by solid state physics.
- An early model of electrical conduction was the Drude model.

It applied kinetic theory to the electrons in a solid.

- What are its main assumptions?
- What are the physical experimental results that could be explained by Drude's model?
- And what results it had overestimated?

Free Electron Theory

- Arnold Sommerfeld combined the classical Drude model with quantum mechanics in:

the free electron model (or Drude-Sommerfeld model).

What are its main assumptions?

- The free electron model gave improved predictions for the heat capacity of metals, however, it was unable to explain the existence of insulators.

Free Electron Fermi Gas

- From many types of experiments

: it was found that a conduction electron in a metal can move freely in a straight path over many atomic distances, undeflected by collisions with other conduction electrons or by collisions with the atom cores.

- In a very pure specimen at low temperatures the mean free path may be as long as 10^8 interatomic spacings (more than 1 cm).

Free Electron Fermi Gas

- **Why is a condensed matter so transparent to conduction electrons ?**
- **The answer is:**
- **1. A conduction electron is not deflected by ion cores arranged on a periodic lattice because matter waves propagate freely in a periodic structure.**
- **2. A conduction electron is scattered only by other conduction electrons as a consequence of Pauli exclusion principle.**
- **By a free electron Fermi gas we mean a gas of free electrons subject to Pauli principle.**

Free Levels in One Dimension

Consider a free electron gas in one dimension, taking account of quantum theory and of the Pauli principle. An electron of mass m is confined to a length L by infinite barriers (Fig. 2). The wavefunction $\psi_n(x)$ of the electron is a solution of the Schrödinger equation $\mathcal{H}\psi = \epsilon\psi$; with the neglect of potential energy we have $\mathcal{H} = p^2/2m$, where p is the momentum. In quantum theory p may be represented by $-i\hbar\, d/dx$, so that

$$\mathcal{H}\psi_n = -\frac{\hbar^2}{2m}\frac{d^2\psi_n}{dx^2} = \epsilon_n\psi_n \ , \qquad (1)$$

where ϵ_n is the energy of the electron in the orbital.

Free Levels in One Dimension

We use the term orbital to denote a solution of the wave equation for a system of only one electron. The term allows us to distinguish between an exact quantum state of the wave equation of a system of *N* electrons and an approximate quantum state which we construct by assigning the *N* electrons to *N* different orbitals, where each orbital is a solution of a wave equation for one electron. The orbital model is exact only if there are no interactions between electrons.

The wavefunction and the energy

The boundary conditions are $\psi_n(0) = 0$; $\psi_n(L) = 0$, as imposed by the infinite potential energy barriers. They are satisfied if the wavefunction is sinelike with an integral number n of half-wavelengths between 0 and L:

$$\psi_n = A \sin\left(\frac{2\pi}{\lambda_n}x\right) , \qquad \tfrac{1}{2}n\lambda_n = L , \tag{2}$$

where A is a constant. We see that (2) is a solution of (1), because

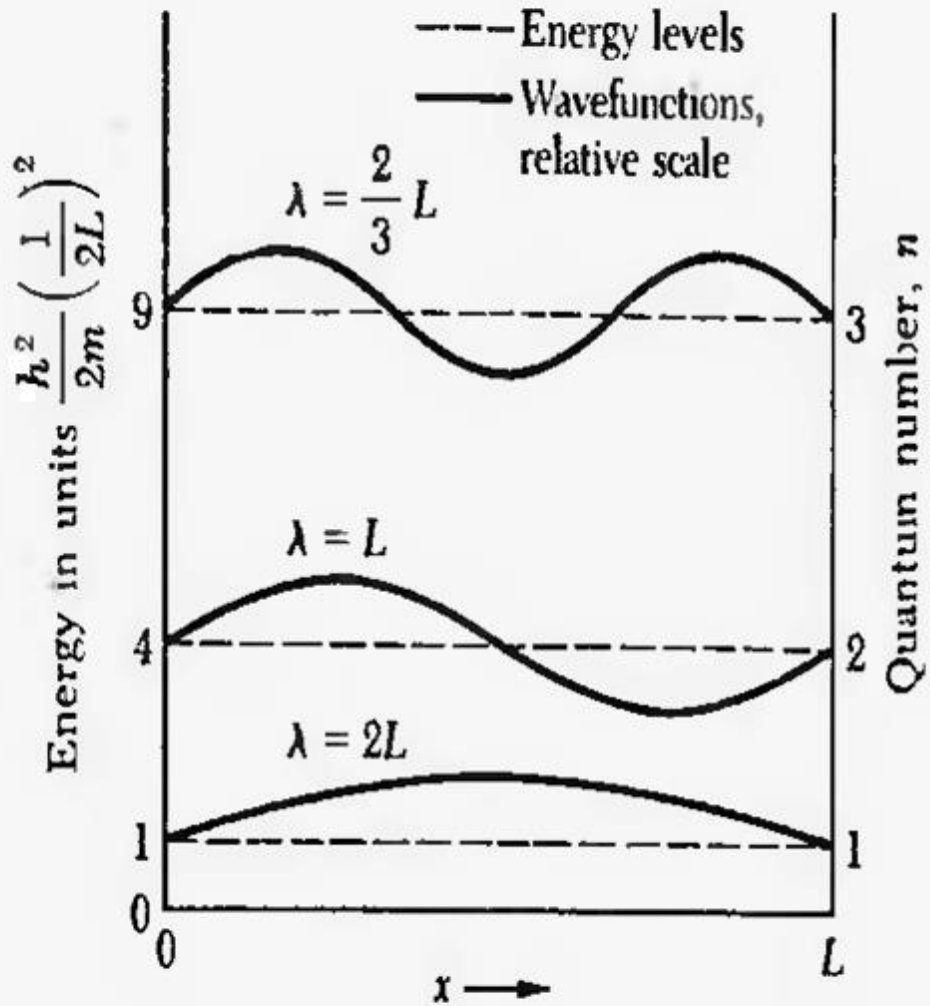
$$-\frac{d\psi_n}{dx} = A\left(\frac{n\pi}{L}\right)\cos\left(\frac{n\pi}{L}x\right) ; \qquad \frac{d^2\psi_n}{dx^2} = -A\left(\frac{n\pi}{L}\right)^2 \sin\left(\frac{n\pi}{L}x\right) ,$$

whence the energy ϵ_n is given by

$$\epsilon_n = \frac{\hbar^2}{2m}\left(\frac{n\pi}{L}\right)^2 . \qquad (3$$

The wavefunction and the energy

Figure 2 First three energy levels and wavefunctions of a free electron of mass m confined to a line of length L. The energy levels are labeled according to the quantum number n which gives the number of half-wavelengths in the wavefunction. The wavelengths are indicated on the wavefunctions. The energy ϵ_n of the level of quantum number n is equal to $(\hbar^2/2m)(n\pi/L)^2$.



- **In a linear solid, the quantum numbers are n and m_s only. Why?**

- **The Fermi energy ε_f :Definition and equation.**

- **The ground state**

- **Fermi-Dirac distribution**

Let n_F denote the topmost filled energy level, where we start filling the levels from the bottom ($n = 1$) and continue filling higher levels with electrons until all N electrons are accommodated. It is convenient to suppose that N is an even number. The condition $2n_F = N$ determines n_F, the value of n for the uppermost filled level.

The **Fermi energy** ϵ_F is defined as the energy of the topmost filled level in the ground state of the N electron system. By (3) with $n = n_F$ we have in one dimension:

$$\epsilon_F = \frac{\hbar^2}{2m}\left(\frac{n_F \pi}{L}\right)^2 = \frac{\hbar^2}{2m}\left(\frac{N\pi}{2L}\right)^2 \qquad (4)$$

The ground state is the state of the N electron system at absolute zero.

The kinetic energy of the electron gas increases as the temperature is increased: some energy levels are occupied which were vacant at absolute zero, and some levels are vacant which were occupied at absolute zero (Fig. 3). The **Fermi-Dirac distribution** gives the probability that an orbital at energy ϵ will be occupied in an ideal electron gas in thermal equilibrium:

$$\boxed{f(\epsilon) = \frac{1}{\exp[(\epsilon - \mu)/k_B T] + 1}} \quad . \tag{5}$$

The quantity μ is a function of the temperature; μ is to be chosen for the particular problem in such a way that the total number of particles in the system comes out correctly—that is, equal to N. At absolute zero $\mu = \epsilon_F$, because in the limit $T \to 0$ the function $f(\epsilon)$ changes discontinuously from the value 1 (filled) to the value 0 (empty) at $\epsilon = \epsilon_F = \mu$. *At all temperatures* $f(\epsilon)$ *is equal to* $\frac{1}{2}$ *when* $\epsilon = \mu$, for then the denominator of (5) has the value 2.

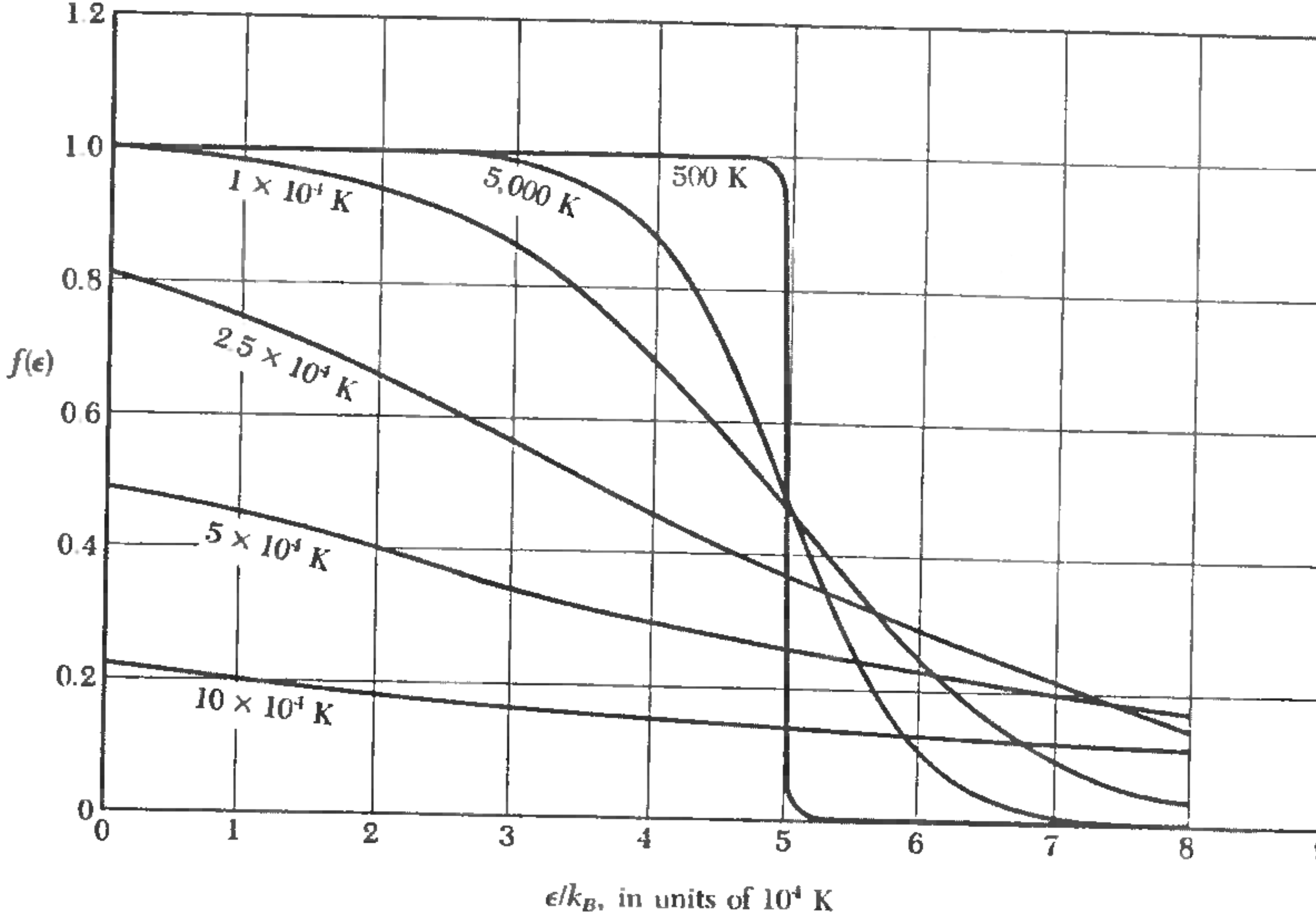


Figure 3 Fermi-Dirac distribution function at various temperatures, for $T_F \equiv \epsilon_F/k_B = 50{,}000$ K. The results apply to a gas in three dimensions. The total number of particles is constant, independent of temperature. The chemical potential at each temperature may be read off the graph as the energy at which $f = 0.5$.

Questions

* **1. What is Solid State Physics all about?**
* **2. Solid materials are formed from densely-packed atoms, which interact intensely, what are the direct results of these interactions?**
* **3:What are the main parameters which make a material crystalline or non-crystalline ?**
* **4: What is Drude model; write its main assumptions.**
* **5: Why a conduction electron in a metal can move freely in straight path over many interatomic spacings undeflected by collisions as if the matter is transparent to it?**
* **6: Find the wavefunction and the energy of a free electron of mass m confined at a length L by infinite barriers. Then deduce the Fermi energy ε_f in an N electron system.**

Internet and Text Book Research

Define shortly Fermi – Dirac Statistics.

What are Fermions and Bosons?

Thank You Very Much