Lectures on Electromagnetic theory I

PH 2151

Lecture 8

Potential Gradient ,the dipole and energy stored in an electrostatic field .

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Potential gradient

Since V=- $\int E.dl$

dv= -E.dl = -$E_x$dx –$E_y$dy –$E_z$dz

E = - $(\frac{\partial V}{\partial x} \; ax + \frac{\partial y}{\partial x} a_y + \frac{\partial V}{\partial z} a_z)$

E=-∇V=-grad V

$$\nabla V= \frac{\partial V}{\partial x} a_x + \frac{\partial V}{\partial y} a_y + \frac{\partial y}{\partial x} a_z \qquad (cartesian)$$

$$\nabla V= \frac{\partial V}{\partial \rho} a_\rho + \frac{1}{\rho} \frac{\partial V}{\partial \varphi} a_\varphi + \frac{\partial V}{\partial z} a_z \qquad (cylindrical)$$

$$\nabla V= \frac{\partial V}{\partial r} a_r + \frac{1}{r} \frac{\partial V}{\partial \theta} a_\theta + \frac{1}{rsin\theta} \frac{\partial V}{\partial \Phi} a_\Phi \quad \text{(spherical)}$$

The operation on V by which -E is obtained is known as the gradient .

The relation between E and V

since $\frac{dV}{dl} = -Ecos\theta$

dV_{max} Will occur when dL is in the direction opposite to E

$\frac{dV}{dl}|_{max} = |E|$

1)The magnitude of E is given by the maximum value of the rate of change of the potential with distance.

2)this is obtained when the direction of E is opposite to the direction of increasing the potential .

The relationship between V as a scalar field and E as a vector field

If a_N is a unite vector normal to the equipotential surface and directed toward the higher potential i.e opposite to the direction of E.

$$E=-\frac{dV}{dL}\Big|_{\max} \; a_N$$

$$\frac{dV}{dL}\Big|_{max} = \frac{dV}{dN}$$

$$E=-\frac{dV}{dN}a_N$$

The electric dipole:

The dipole has moment $P = P.a_r = Qd\cos\theta$

potential field of electric dipole $V = \frac{P.a_r}{4\pi\varepsilon_o r^2} = \frac{|P|\cos\theta}{4\pi\varepsilon_o r^2}$

The potential field in general form is given by

$v = \frac{1}{4\pi\epsilon 0|r-r\prime|^2}\,p.\frac{r-r\prime}{|r-r\prime|}$

The electric field intensity $E = \frac{Qd}{4\pi\epsilon_0 r^3}(2\cos\theta\, a_r + \sin\theta\, a_\theta)$

Energy stored in an electrostatic field W_E

For discrete charge distribution

$$W_E = \frac{1}{2}(Q_1V_1 + Q_2V_2 + Q_3V_3 + \ldots\ldots) = \frac{1}{2}\sum_{m=1}^{m=N} Q_mV_m$$

For continuous discharge distribution

$$W_E = \frac{1}{2}\int_{Vol} D.E dv = \frac{1}{2}\int_{Vol} \varepsilon_0 E^2 dv$$

Example :Calculate the energy stored and the energy density in the electrostatic field of a section of coaxial cable of length L and the surface charge density on the inner conductor whose radius a is ρ_s

Problems

1)Three identical point charges of 4pc each located at the corners of an equilateral triangle 0.5 mm on each side in free space. How much work must be done to move one charge to a point equidistant from the other two and on the line joining them .

2)Given the potential field $V=2x^2y -5z$,find at the point P(-4,3,6):

a)The potential V. b)The field intensity E

c)The direction of E and a_N,where a_N unit vector directed towards the higher potentials.

d) $\left.\frac{dV}{dL}\right|_{\max}$ e) The electric flux density D

3)A dipole moment $P=6a_z$ nc.m is located at the origin .Find V at P ($r=4m, \theta=20^o$, $\varphi=0$) and E at the same point P.

4)Four 0.8nc point charges are located in free space at the corners of a square 4 cm on a side. Find the energy stored .

5)Find the energy stored in free space for the region $2mm< r < 3mm$, $0<\theta<90$, $0<\varphi <90$, given the potential

a) $V=\frac{200}{r}$

b) $V=\frac{300 cos\theta}{r^3}$