Lecture (1) on classical electrodynamics I

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Introduction

<u>Text Books:</u>

- **Sadiku, *Elements of Electromagnetics, Oxford University.***
- ***Griffiths, Introduction to Electrodynamics, Prentice Hall.***
- ***Jackson, Classical Electrodynamics,* New York: John Wiley & Sons.**
- ***Open sources: MIT open courses, Salman bin Abdelaziz Univ. ….***

<u>Evaluation will be done through:</u>

- ***<u>Final Written exam</u>***
- ***<u>Oral exam</u>***
- ***<u>Quizzes during the lecture, you need your tools (pencile, papers, calculator)</u>***
- ***<u>Homework</u>***

Keywords

- **Vector analysis, Coordinate systems, and transformations**

- ***Electric field in materials, Polarization in dielectric materials, Continuity equation, Relaxation time, Boundary conditions***

- ***Electrostatic boundary value problems, variable separation, method of images***

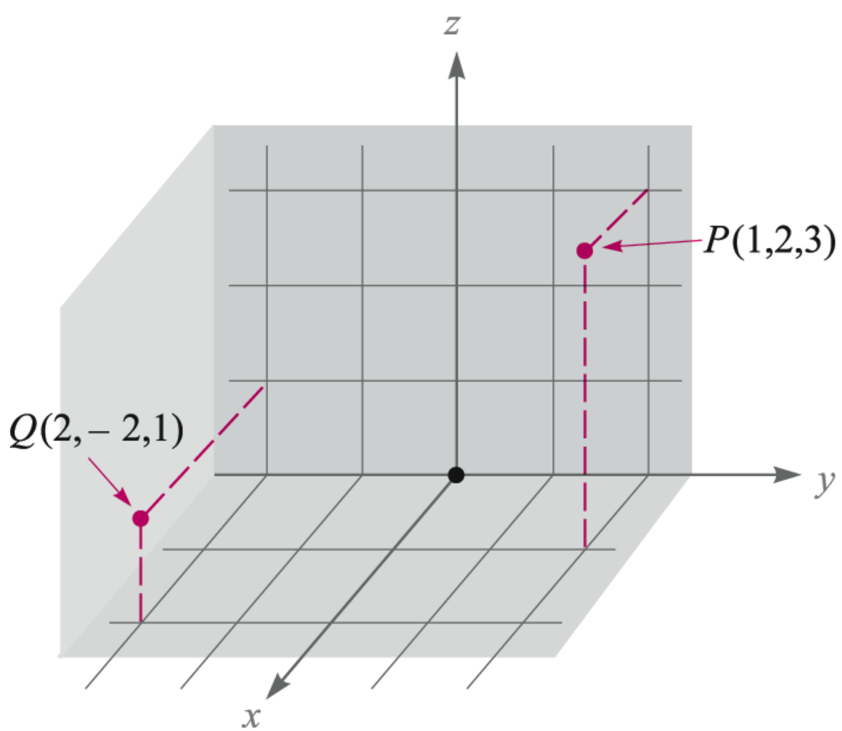
- ***Magnetic fields: Bio-Savart's law and Problems, Ampere's law and problems, Analogy between electric and magnetic fields***

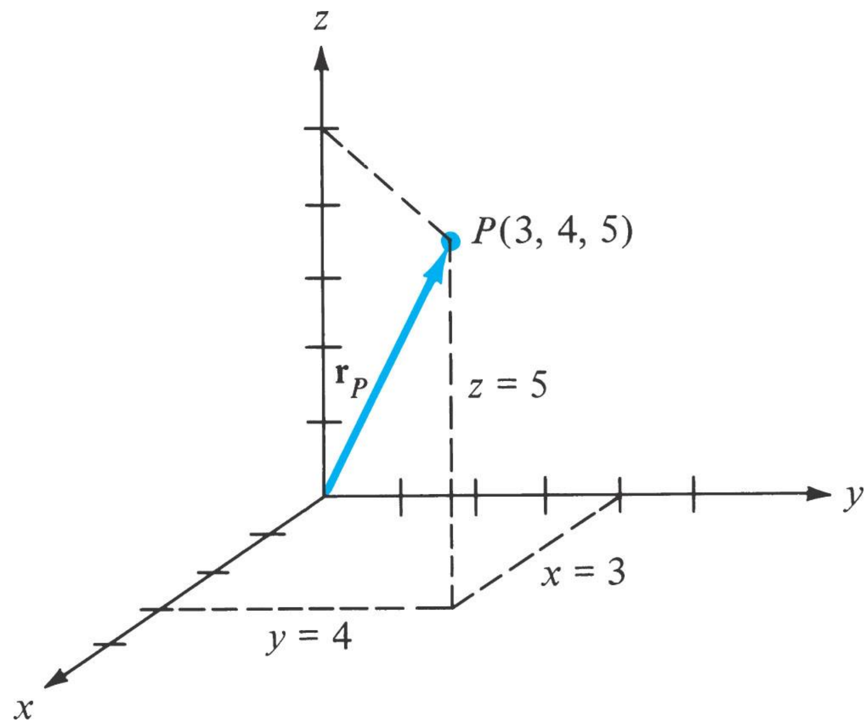
- ***Maxwell's equations in vacuum and matter, Problems***

Cartesian coordinates & Vector analysis

Rectangular Coordinate System

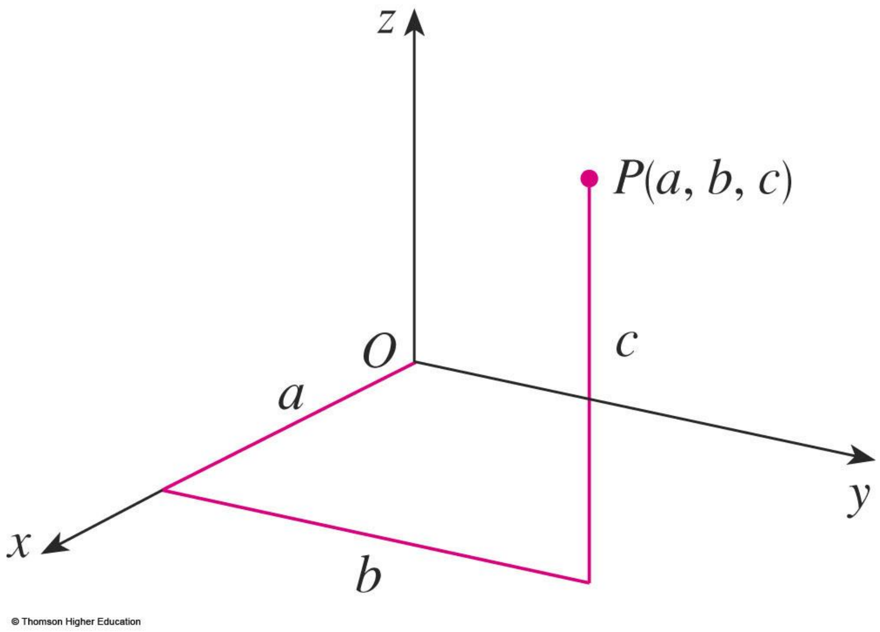
- **Each point in 3D space has three components on x, y, and z axes.**





Rectangular Coordinate System

- **Each point in 3D space has three components on x, y, and z axes.**



$$\vec{P} = a\ \vec{a}_x + b\ \vec{a}_y + c\ \vec{a}_z$$

$$\mathbf{P} = a\ \mathbf{a}_x + b\ \mathbf{a}_y + c\ \mathbf{a}_z$$

- **The distance from the origin to**

The point P is

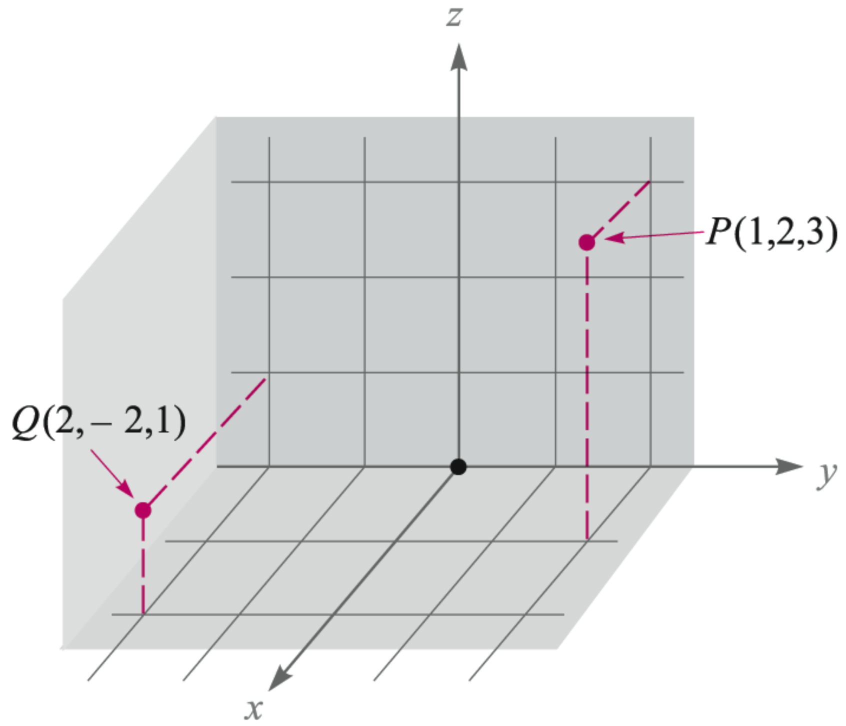
$$|\mathbf{P}| = \sqrt{a^2 + b^2 + c^2}$$

The unit vector along $\vec{OP} = \mathbf{OP}$ **is**

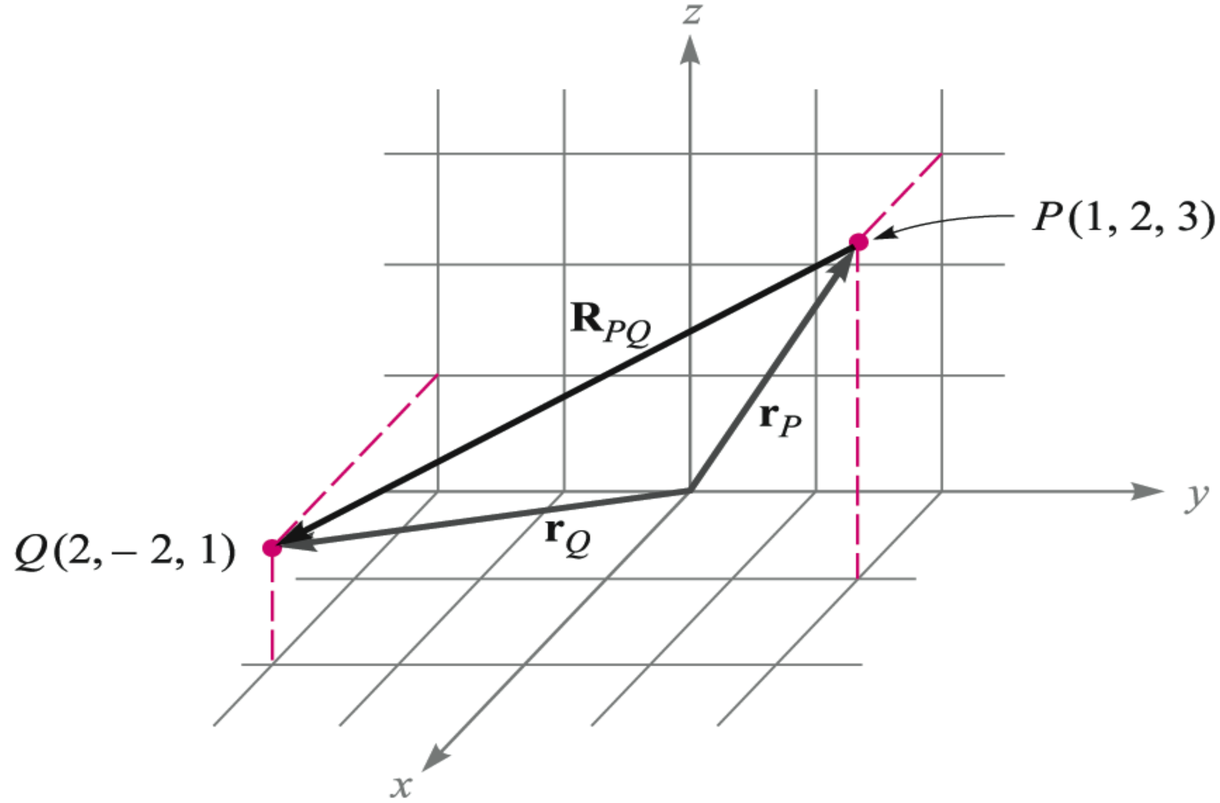
$$\frac{\mathbf{P}}{|\mathbf{P}|}$$

Quiz 1

- **Assume two charges at the points P and Q. Calculate the distance between the tow charges and the unit vector from P to Q. Find the electric force between them if they are tow electrons and the electric field at the origin. Revise previous lectures in grade 1 and 2.**



Hint:



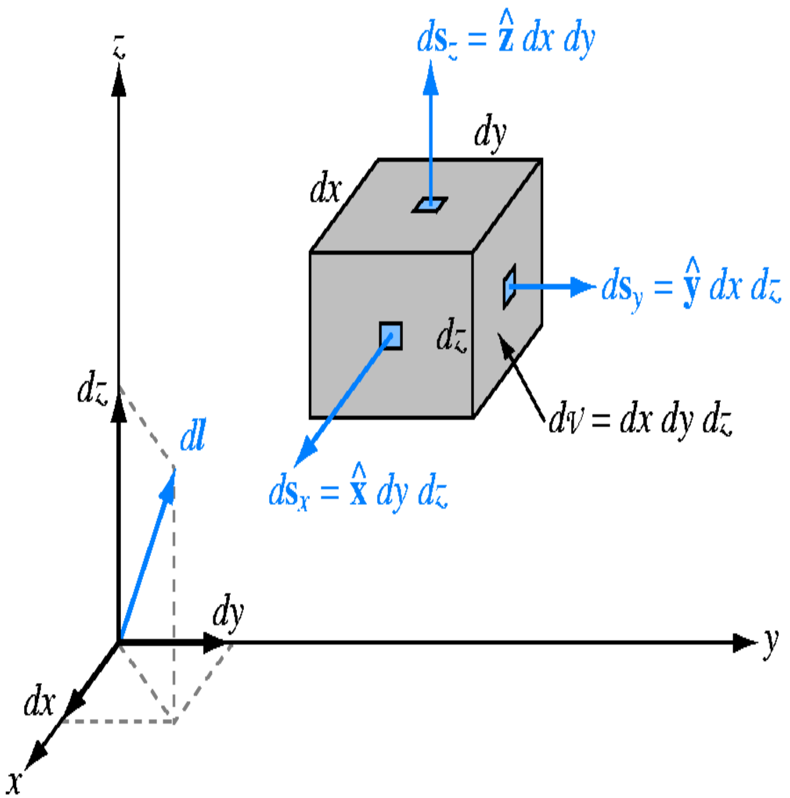
$$\mathbf{R}_{PQ} = \mathbf{r}_Q - \mathbf{r}_P = (2-1)\mathbf{a}_x + (-2-2)\mathbf{a}_y + (1-3)\mathbf{a}_z$$
$$= \mathbf{a}_x - 4\mathbf{a}_y - 2\mathbf{a}_z$$

Differential Volume Element

- The displacement from point to point is a vector quantity.

$$\mathbf{dl} = dx\ \mathbf{a}_x + dy\ \mathbf{a}_y + dz\ \mathbf{a}_z$$

- The surface is a vector quantity. The surface vector of any surface is the magnitude of the surface area in the direction of the unit vector normal to that surface
- Volume element is a scaler quantity $$dV = dx\,dy\,dz$$



Example

Specify the unit vector extending from the origin toward the point $G(2, -2, -1)$

$$\mathbf{G} = 2\mathbf{a}_x - 2\mathbf{a}_y - \mathbf{a}_z$$

$$|\mathbf{G}| = \sqrt{(2)^2 + (-2)^2 + (-1)^2} = 3$$

$$\mathbf{a}_G = \frac{\mathbf{G}}{|\mathbf{G}|} = \frac{2}{3}\mathbf{a}_x - \frac{2}{3}\mathbf{a}_y - \frac{1}{3}\mathbf{a}_z = \underline{0.667\mathbf{a}_x - 0.667\mathbf{a}_y - 0.333\mathbf{a}_z}$$

Vector Field

We are accustomed to thinking of a specific vector:

$$\mathbf{v} = v_x\mathbf{a}_x + v_y\mathbf{a}_y + v_z\mathbf{a}_z$$

A vector field is a *function* defined in space that has magnitude and direction at all points:

$$\mathbf{v}(\mathbf{r}) = v_x(\mathbf{r})\mathbf{a}_x + v_y(\mathbf{r})\mathbf{a}_y + v_z(\mathbf{r})\mathbf{a}_z$$

where $\mathbf{r} = (x,y,z)$

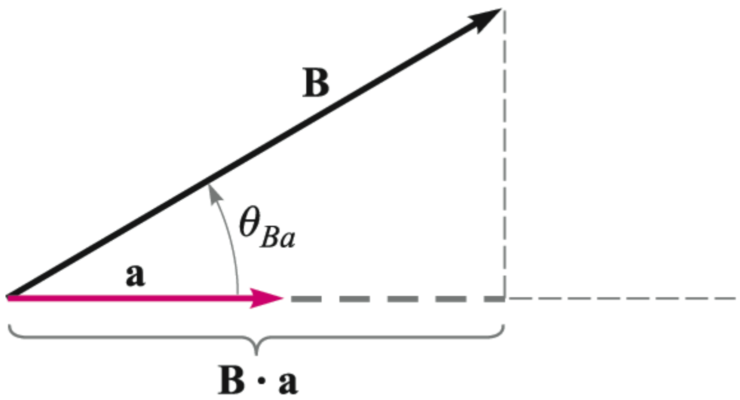
The Dot Product

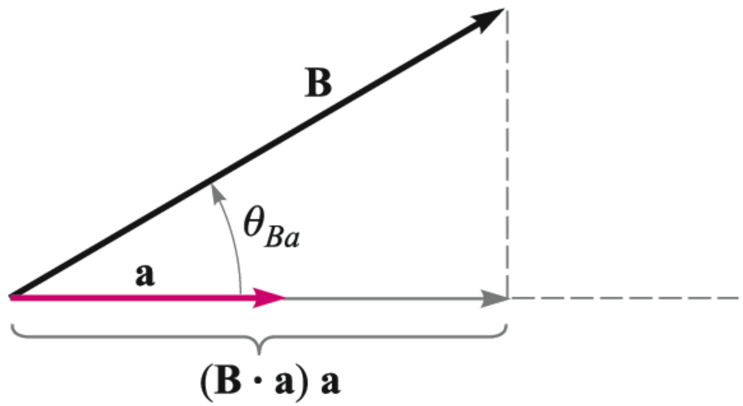
Given two vectors **A** and **B**, the *dot product*, or *scalar product*, is defined as the product of the magnitude of **A**, the magnitude of **B**, and the cosine of the smaller angle between them,

$$\boxed{\mathbf{A} \cdot \mathbf{B} = |\mathbf{A}|\,|\mathbf{B}| \cos \theta_{AB}}$$

Commutative Law: $\boxed{\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}}$

Vector Projections Using the Dot Product





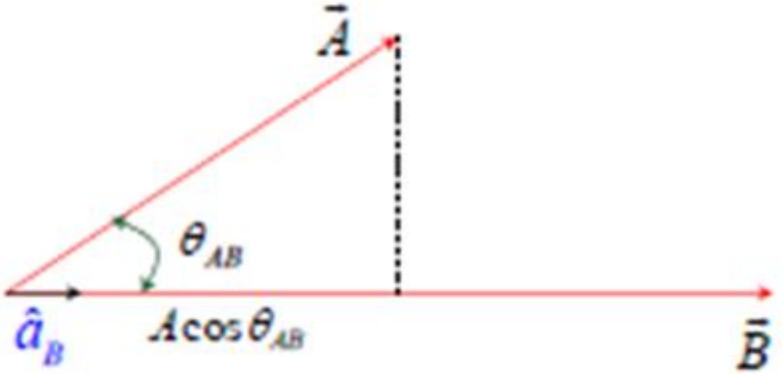
B • a gives the component of **B** in the horizontal direction

(**B • a**) **a** gives the *vector* component of **B** in the horizontal direction

Projection of a vector on another vector

- Scalar component of $\vec{A}$ along $\vec{B}$ is called projection of $\vec{A}$ on $\vec{B}$ and is given by

$$A_B = A\cos\theta_{AB}$$
$$= \left|\vec{A}\right|\left|\hat{a}_B\right|\cos\theta_{AB}$$
$$= \vec{A}\cdot\hat{a}_B$$



Operational Use of the Dot Product

Given $\begin{cases} \mathbf{A} = A_x \mathbf{a}_x + A_y \mathbf{a}_y + A_z \mathbf{a}_z \\ \mathbf{B} = B_x \mathbf{a}_x + B_y \mathbf{a}_y + B_z \mathbf{a}_z \end{cases}$

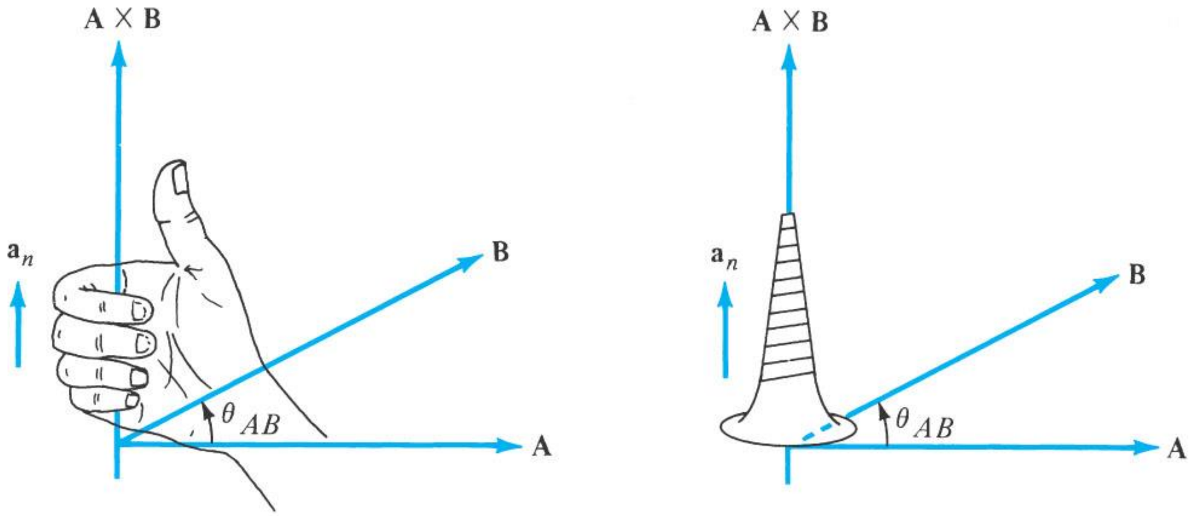
Find $\boxed{\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z}$

where we have used: $\begin{cases} \mathbf{a}_x \cdot \mathbf{a}_y = \mathbf{a}_y \cdot \mathbf{a}_z = \mathbf{a}_x \cdot \mathbf{a}_z = 0 \\ \mathbf{a}_x \cdot \mathbf{a}_x = \mathbf{a}_y \cdot \mathbf{a}_y = \mathbf{a}_z \cdot \mathbf{a}_z = 1 \end{cases}$

Note also: $\boxed{\mathbf{A} \cdot \mathbf{A} = A^2 = |\mathbf{A}|^2}$

Cross Product

The cross product $\mathbf{A} \times \mathbf{B}$ is a vector; the magnitude of $\mathbf{A} \times \mathbf{B}$ is equal to the product of the magnitudes of $\mathbf{A}$, $\mathbf{B}$, and the sine of the smaller angle between $\mathbf{A}$ and $\mathbf{B}$; the direction of $\mathbf{A} \times \mathbf{B}$ is perpendicular to the plane containing $\mathbf{A}$ and $\mathbf{B}$ and is along that one of the two possible perpendiculars which is in the direction of advance of a right-handed screw as $\mathbf{A}$ is turned into $\mathbf{B}$.



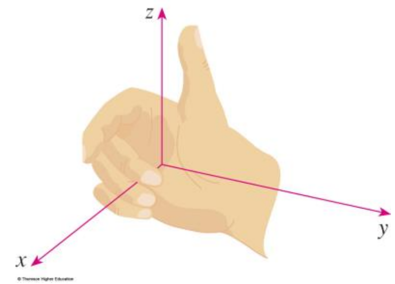
(a) (b)

$$\mathbf{A} \times \mathbf{B} = \mathbf{a}_N |\mathbf{A}|\,|\mathbf{B}| \sin\theta_{AB}$$

Operational Definition of the Cross Product in Rectangular Coordinates

Begin with:
$$\mathbf{A}\times\mathbf{B} = A_xB_x\mathbf{a}_x\times\mathbf{a}_x + A_xB_y\mathbf{a}_x\times\mathbf{a}_y + A_xB_z\mathbf{a}_x\times\mathbf{a}_z$$
$$+ A_yB_x\mathbf{a}_y\times\mathbf{a}_x + A_yB_y\mathbf{a}_y\times\mathbf{a}_y + A_yB_z\mathbf{a}_y\times\mathbf{a}_z$$
$$+ A_zB_x\mathbf{a}_z\times\mathbf{a}_x + A_zB_y\mathbf{a}_z\times\mathbf{a}_y + A_zB_z\mathbf{a}_z\times\mathbf{a}_z$$

$$\boxed{\text{where}\ \begin{cases}\mathbf{a}_x\times\mathbf{a}_y = \mathbf{a}_z\\ \mathbf{a}_y\times\mathbf{a}_z = \mathbf{a}_x\\ \mathbf{a}_z\times\mathbf{a}_x = \mathbf{a}_y\end{cases}}$$



Therefore:

$$\mathbf{A}\times\mathbf{B} = (A_yB_z - A_zB_y)\mathbf{a}_x + (A_zB_x - A_xB_z)\mathbf{a}_y + (A_xB_y - A_yB_x)\mathbf{a}_z$$

Or…
$$\boxed{\mathbf{A}\times\mathbf{B} = \begin{vmatrix}\mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}}$$

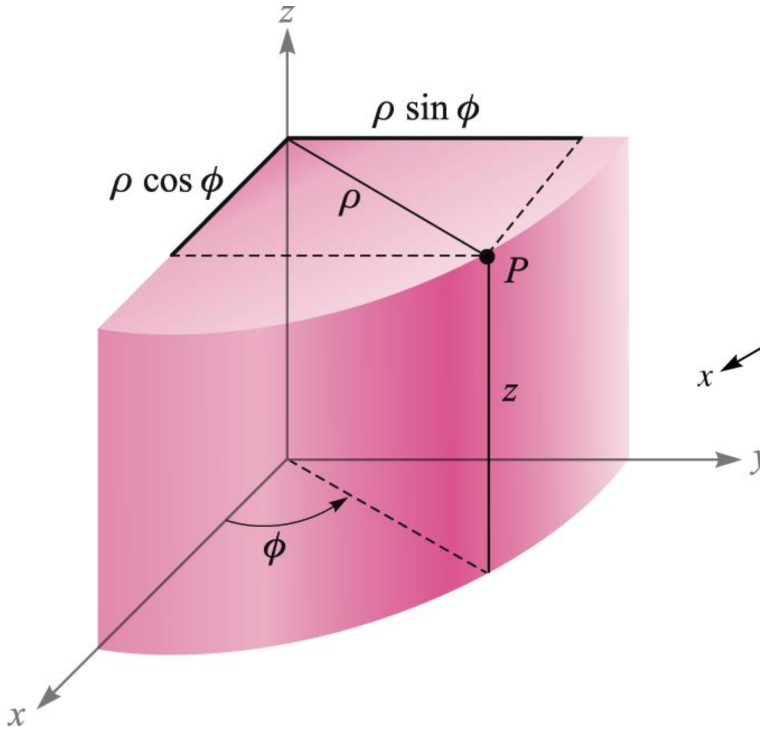
Cylindrical coordinates

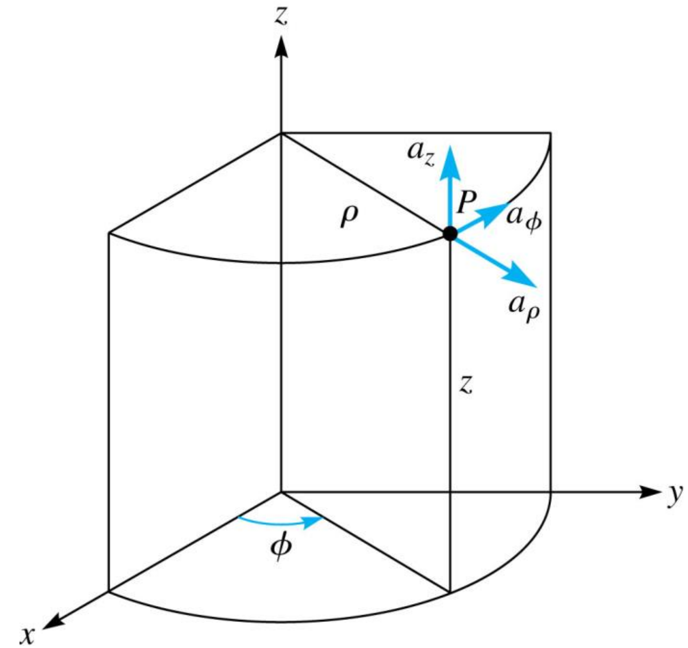
Cylindrical Coordinate Systems

- Any point in space is considered to be at the intersection of three mutually perpendicular surfaces:
  - A circular cylinder (ρ=constant)
  - A vertical plane (Φ=constant)
  - A horizontal plane (z=constant)
- Any point in space is represented by three coordinates $P(\rho,\Phi,z)$
  - ρ denotes the radius of an imaginary cylinder passing through P, or the radial distance from z axis to the point P.
  - Φ denotes azimuthal angle, measured from x axis to a vertical intersecting plane passing through P.
  - z denotes distance from xy-plane to a horizontal intersecting plane passing through P. It is the same as in rectangular coordinate system.

Point Transformations in Cylindrical Coordinates

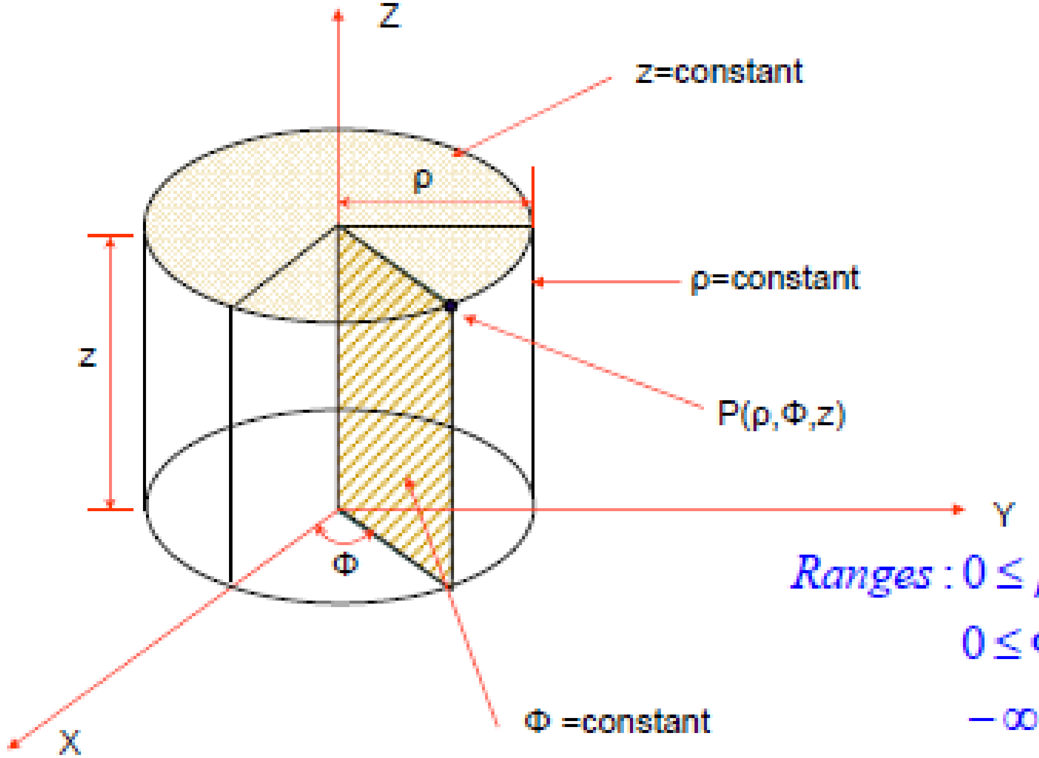
$$\rho = \sqrt{x^2 + y^2} \quad (\rho \geq 0)$$
$$\phi = \tan^{-1} \frac{y}{x}$$
$$z = z$$





$$x = \rho \cos \phi$$
$$y = \rho \sin \phi$$
$$z = z$$

Cylindrical Coordinate Systems



$$Ranges: 0 \le \rho < \infty$$
$$0 \le \Phi < 2\pi$$
$$-\infty < z < \infty$$

$$\mathbf{A} = A_\rho \mathbf{a}_\rho + A_\phi \mathbf{a}_\phi + A_z \mathbf{a}_z$$

$$|\mathbf{A}| = \sqrt{A_\rho^2 + A_\phi^2 + A_z^2}$$

Cylindrical Coordinate Systems

- When a point is displaced in the three dimensions:

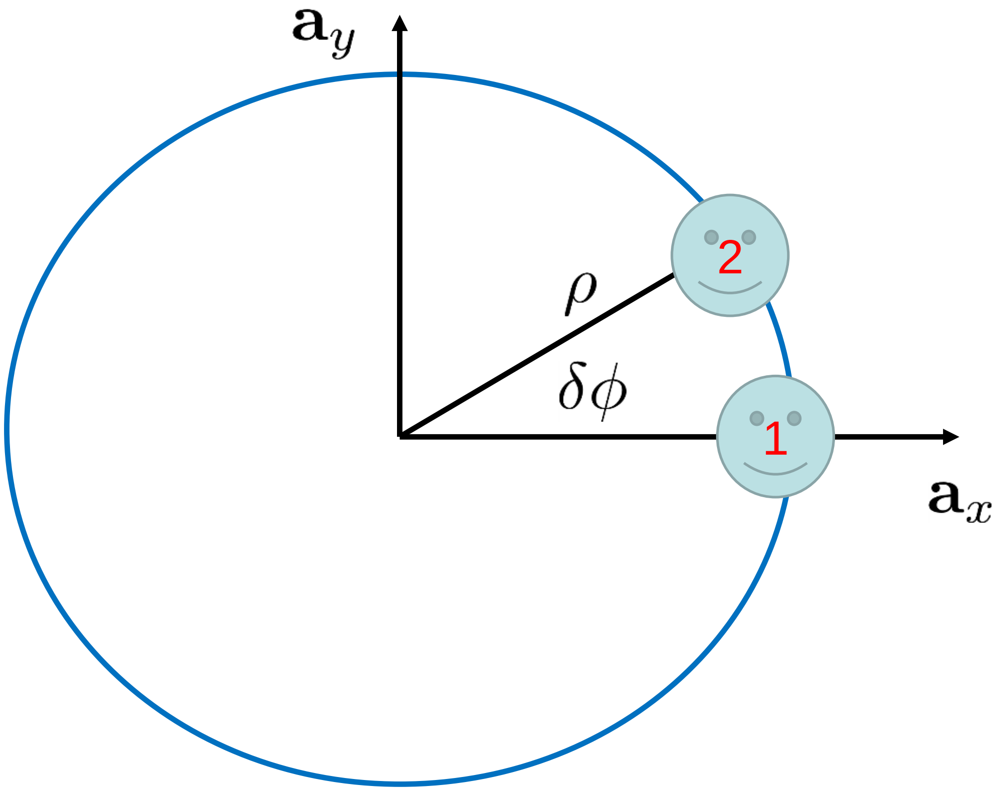
$$d\mathbf{l} = dl_\rho\ \mathbf{a}_\rho + dl_\phi\ \mathbf{a}_\phi + dl_z\ \mathbf{a}_\phi$$

- Note that the displacements along the radius and z-axis are straight lines.
- The displacement along ϕ direction is curvature:

$$dl_\phi = \rho d\phi$$

Therefore;

$$d\mathbf{l} = d\rho\ \mathbf{a}_\rho + \rho d\phi\ \mathbf{a}_\phi + z\ \mathbf{a}_\phi$$



Cylindrical Coordinate Systems surfaces

- **The surface elements when**
  - **ρ is constant**

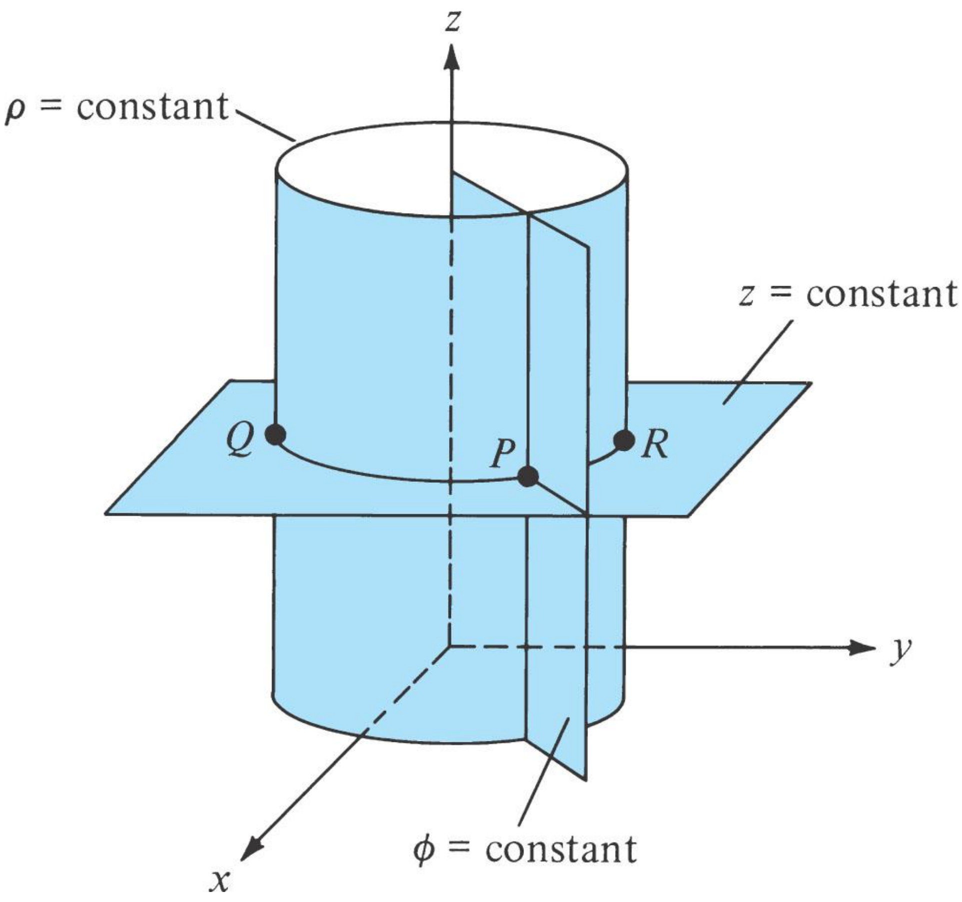
$$ds = \rho d\phi\ dz\ \mathbf{a}_\rho$$

  - **φ is constant**

$$ds = d\rho\ dz\ \mathbf{a}_\phi$$

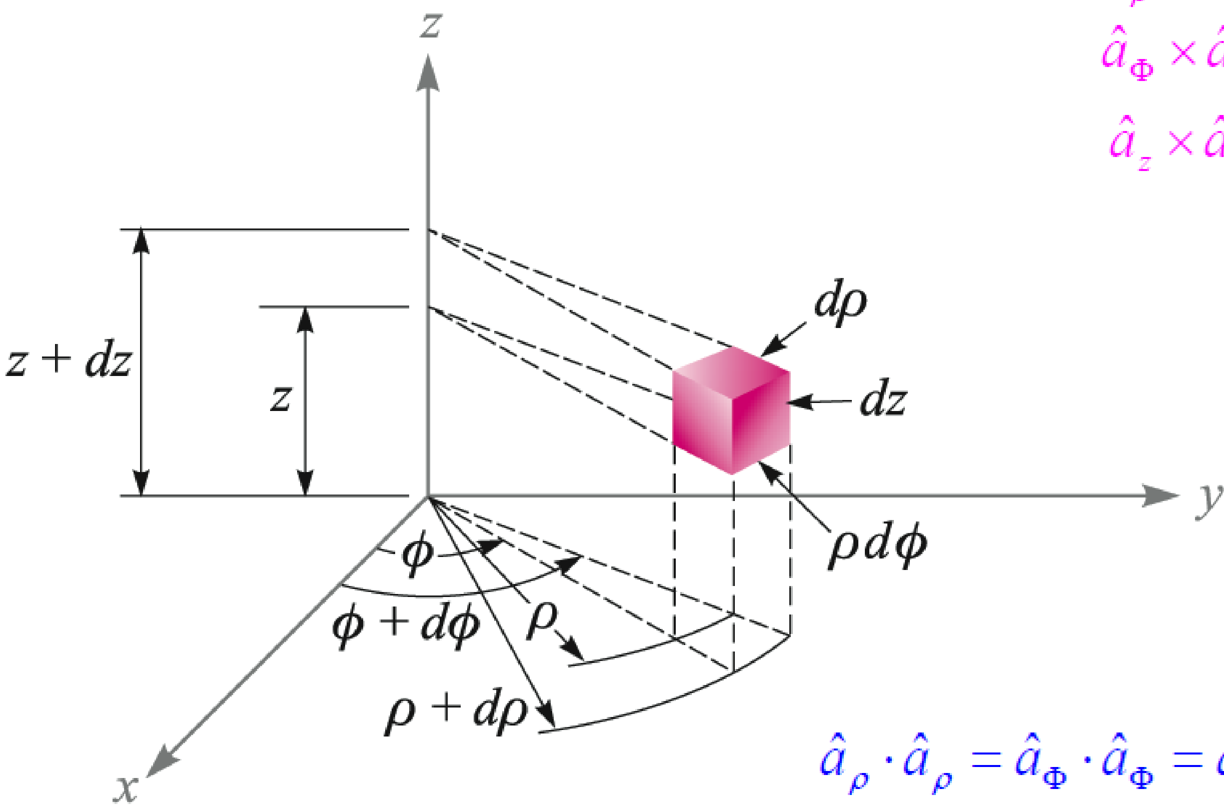
  - **z is constant**

$$ds = d\rho\ \rho d\phi\ \mathbf{a}_z$$



Differential Volume in Cylindrical Coordinates

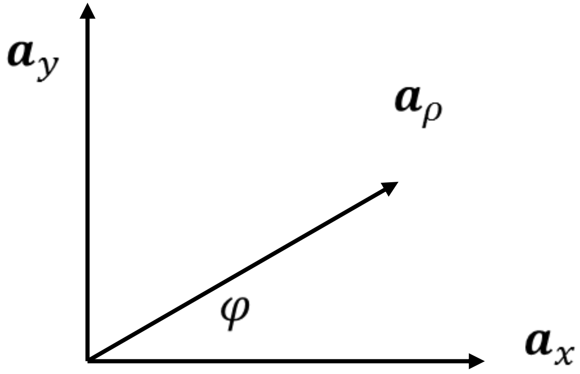
$$\hat{a}_\rho \times \hat{a}_\Phi = \hat{a}_z$$
$$\hat{a}_\Phi \times \hat{a}_z = \hat{a}_\rho$$
$$\hat{a}_z \times \hat{a}_\rho = \hat{a}_\Phi$$



$$\hat{a}_\rho \cdot \hat{a}_\rho = \hat{a}_\Phi \cdot \hat{a}_\Phi = \hat{a}_z \cdot \hat{a}_z = 1$$
$$\hat{a}_\rho \cdot \hat{a}_\Phi = \hat{a}_\Phi \cdot \hat{a}_z = \hat{a}_z \cdot \hat{a}_\rho = 0$$

$$\boxed{dV = \rho\, d\rho\, d\phi\, dz}$$

Dot Products of Unit Vectors in Cylindrical and Rectangular Coordinate Systems



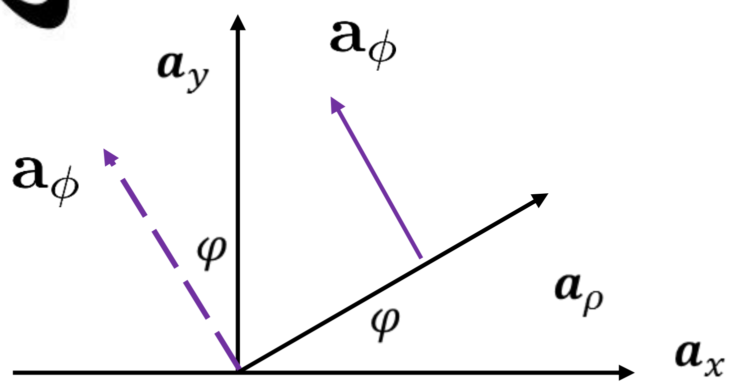
$$\mathbf{a}_\rho = cos(\phi)\,\mathbf{a}_x + sin(\phi)\,\mathbf{a}_y$$

Table 1.1 Dot products of unit vectors in cylindrical and rectangular coordinate systems

	$\mathbf{a}_\rho$	$\mathbf{a}_\phi$	$\mathbf{a}_z$
$\mathbf{a}_x \cdot$	$\cos\phi$	$-\sin$	0
$\mathbf{a}_y \cdot$	$\sin\phi$	$\cos\phi$	0
$\mathbf{a}_z \cdot$	0	0	1

Look at page 4: relations 2.9 and 2.10 in the given manuscript and figure 2.3

Dot Products of Unit Vectors in Cylindrical and Rectangular Coordinate Systems



$$\mathbf{a}_\phi = -sin(\phi)\,\mathbf{a}_x + cos(\phi)\,\mathbf{a}_y$$

Table 1.1 Dot products of unit vectors in cylindrical and rectangular coordinate systems

	$\mathbf{a}_\rho$	$\mathbf{a}_\phi$	$\mathbf{a}_z$
$\mathbf{a}_x \cdot$	$\cos\phi$	$-\sin$	0
$\mathbf{a}_y \cdot$	$\sin\phi$	$\cos\phi$	0
$\mathbf{a}_z \cdot$	0	0	1

Look at page 4: relations 2.9 and 2.10 in the given manuscript and figure 2.3

Example (1)

Transform the vector, $\mathbf{B} = y\mathbf{a}_x - x\mathbf{a}_y + z\mathbf{a}_z$ into cylindrical coordinates:

Start with: $B_\rho = \mathbf{B} \cdot \mathbf{a}_\rho = y(\mathbf{a}_x \cdot \mathbf{a}_\rho) - x(\mathbf{a}_y \cdot \mathbf{a}_\rho)$

$$B_\phi = \mathbf{B} \cdot \mathbf{a}_\phi = y(\mathbf{a}_x \cdot \mathbf{a}_\phi) - x(\mathbf{a}_y \cdot \mathbf{a}_\phi)$$

Then:

$$\begin{aligned} B_\rho &= \mathbf{B} \cdot \mathbf{a}_\rho = y(\mathbf{a}_x \cdot \mathbf{a}_\rho) - x(\mathbf{a}_y \cdot \mathbf{a}_\rho) \\ &= y\cos\phi - x\sin\phi = \rho\sin\phi\cos\phi - \rho\cos\phi\sin\phi = 0 \end{aligned}$$

$$B_\phi = \mathbf{B} \cdot \mathbf{a}_\phi = y(\mathbf{a}_x \cdot \mathbf{a}_\phi) - x(\mathbf{a}_y \cdot \mathbf{a}_\phi)$$

$$= -y\sin\phi - x\cos\phi = -\rho\sin^2\phi - \rho\cos^2\phi = -\rho$$

Example (1)

Finally:

$$B_\rho = \mathbf{B}\cdot\mathbf{a}_\rho = y(\mathbf{a}_x\cdot\mathbf{a}_\rho) - x(\mathbf{a}_y\cdot\mathbf{a}_\rho)$$

$$= y\cos\phi - x\sin\phi = \rho\sin\phi\cos\phi - \rho\cos\phi\sin\phi = 0$$

$$B_\phi = \mathbf{B}\cdot\mathbf{a}_\phi = y(\mathbf{a}_x\cdot\mathbf{a}_\phi) - x(\mathbf{a}_y\cdot\mathbf{a}_\phi)$$

$$= -y\sin\phi - x\cos\phi = -\rho\sin^2\phi - \rho\cos^2\phi = -\rho$$

$$\boxed{\mathbf{B} = -\rho\mathbf{a}_\phi + z\mathbf{a}_z}$$

Jacobian matrix

- **Convert from Cylindrical into Cartesian coordinates**

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \frac{dx}{d\rho} & \frac{dy}{d\rho} & \frac{dz}{d\rho} \\ \frac{dx}{\rho d\varphi} & \frac{dy}{\rho d\varphi} & \frac{dz}{\rho d\varphi} \\ \frac{dx}{dz} & \frac{dy}{dz} & \frac{dz}{dz} \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\varphi \\ A_z \end{bmatrix} \Rightarrow \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\varphi \\ A_z \end{bmatrix}$$

- **Convert from Cartesian into Cylindrical coordinates**

$$\begin{bmatrix} A_\rho \\ A_\varphi \\ A_z \end{bmatrix} = \begin{bmatrix} \frac{d\rho}{dx} & \frac{\rho d\varphi}{dx} & \frac{dz}{dx} \\ \frac{d\rho}{dy} & \frac{\rho d\varphi}{dy} & \frac{dz}{dy} \\ \frac{d\rho}{dz} & \frac{\rho d\varphi}{dz} & \frac{dz}{dz} \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\varphi \\ A_z \end{bmatrix} \Rightarrow \begin{bmatrix} A_\rho \\ A_\varphi \\ A_z \end{bmatrix} = \begin{bmatrix} \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\varphi \\ A_z \end{bmatrix}$$

Example (1) page 9

- **Given point P(-2,6,3) and vector** $\mathbf{A} = y\,\mathbf{a}_x + (x+z)\mathbf{a}_y$ **, express P and A in cylindrical coordinates. Evaluate A at P in the Cartesian and Cylindrical coordinates.**

Example (1) page 9

Homework

- **Exercise 2.1 page 11:**

- **Time to get full mark of the exercise is next week. (*otherwise you will get half of the points*)**