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# THE CHARACTERISTIC OF A SPHERICAL ELECTROSTIC PROBE IMMERSED IN A QUISCENT DENSE PLASMA ACTED UPON BY A MAGNETIC FIELD.

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## ABSTRACT

In This paper, an analytical treatment is developed for solving the problem of a spherical electrostatic probe immersed in plasma. The plasma is dense, weekly ionized and quiescent, and acted upon by a uniform magnetic field. The obtained solution discribes the electric field strength in the plasma due to the existence of a negatively biased electrostatic probe having small potential with respect to the plasma. This solution enables us to obtain the characteristic of the probe, from which the electron temperature and the charge concentration in the plasma can be calculated.

## 1. INTRODUCTION

This problem of a spherical electrostatic probe was treated before ( Bush, William B, Fendell Francis E (19[illegible]) using the cyclotron absorption. The electron temperature and the plasma density were determined ( D. K. Akulina, Yu. V. Knol'nov (1978), I. H. Hutchinson, D. S. Komm (1977) and J. Hosea, V. Arunasalon, R. Cano (1977)). The charge concentration in the main region was determined before due to the existence of an electrostatic probe in the AKL M. Y. (1992)). The potential distribution in the main region was also determined before for the same case. ( Abdel Aziz- M. A., Sabbah A. S., Shehata Ph. and AKL M. Y (1994)). The characteristic was determined analytically for a high dense quiescent plasma ( Abdel-Aziz, M. A. (1977)). Sanmartin had discussed the problem of a probe in a fully ionised plasma in a strong magnetic field ( Sanmartin

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# خاصية مجس كروى كهروستاتيكى موجود فى بلازما كثيفة ساكنة مع مجال مغناطيسى

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فى هذا البحث ، تم إستعمال طريقة تحليلية لحل مشكلة مجس كروى كهروستاتيكى مغموس فى بلازما كثيفة ساكنة ضعيفة التأين فى وجود مجال مغناطيسى ضعيف منتظم . تصف الحلول التى تم الحصول عليها شدة المجال الكهربى لبلازما نتيجة لوجود مجس كروى كهروستاتيكى ذو جهد صغير بالنسبة للبلازما ومعرض لمجال مغناطيسى منتظم . تمكننا هذه الحلول من الحصول على خاصية المجس ، والتى منها يمكن حساب خواص البلازما مثل درجة حرارة الالكترونات وتركيز الشحنات فيها .

(1967)) and ( Sanmartin (1970)). Cohen extended the continuum electrostatic probe theory for a slightly ionised, collision-dominated plasma to allow a uniform magnetic field ( Choen (1969)). Kalyan treated the same problem, but he used numerical solutions ( Kalyan k., and Cohen (1972)). In the recent case an analytical treatment is used for solving the differential equations of the problem of a spherical electrostatic probe, immersed in a quiscent, dense plasma due to the existence of a uniform magnetic field.

**Method of Analysis:-**

The equations of continuum mechannics describing the problem can be written in the following form (Kalyan K., Niyogi and Ira Cohen (1972)):-

$$\vec{\nabla}.\vec{\Gamma}_{\pm} = 0 \qquad (1)$$

where :-

$$\vec{\Gamma}\pm = -\vec{\nabla}.(D_{\pm}N_{\pm}) \pm \mu\,\vec{E}\,N_{\pm} \qquad (2)$$

$$\vec{\nabla}.\vec{E} = 4\pi e(N_{+} - N_{-}) \qquad (3)$$

Writting equations (1) and (3) in the dimensionless form :-

$$\frac{\partial^2 n_{\pm}}{\partial x^2} + \frac{\partial^2 n_{\pm}}{\partial y^2} + (1+\Omega_{\pm}^2)\frac{\partial^2 n_{\pm}}{\partial z^2}$$

$$\pm\frac{1}{2}[(\tau+1)\pm(\tau-1)]x\Bigg[\frac{\partial}{\partial x}(n_{\pm}\frac{\partial\Psi}{\partial x}) +$$

$$+\frac{\partial}{\partial y}(n_{\pm}\frac{\partial\Psi}{\partial y}) + (1+\Omega_{\pm}^2)\frac{\partial}{\partial z}(n_{\pm}\frac{\partial\Psi}{\partial z}) +$$

$$+\Omega_{\pm}^2\left(\frac{\partial n_{\pm}}{\partial x}\frac{\partial\Psi}{\partial y} - \frac{\partial n_{\pm}}{\partial y}\frac{\partial\Psi}{\partial x}\right)\Bigg] = 0 \qquad (4)$$

and,

$$\frac{\partial^2\Psi}{\partial x^2}+\frac{\partial^2\Psi}{\partial y^2}+\frac{\partial^2\Psi}{\partial z^2}=\rho_p^{-2}(n_- - n_+) \tag{5}$$

where :-

$\vec{E}$ = The electric field strength

$\tau$ = The ratio of the electreon remperature to ion temperature

$n_+$ = The ionic number density

$n_-$ = The electronic number density

$\Psi$ = The dimensionless plasma potential

$\vec{J}_+$ = The dimensionless ionic current density

$\vec{J}_-$ = The dimensionless electronic current density

$\Omega$ = The ratio of cyclotron freqency to collision frequency.

$$=\frac{\omega_b}{\nu_c}$$

$\omega_b$ = The cyclotron frequency

$$=\frac{eB}{mc}$$

$r_P$ =The radius of the probe

$\lambda_D$ =The Debye length

Equations (4) and (5) are subjected to the following boundary conditions:

$$n_+ = n_- = 1 \text{ as } r \to \infty$$

$$\Psi \to 0 \qquad \text{as } r \to \infty$$

equations (4) and (5) were solved before in the quasineutral region ( Abdel-Aziz, M. A, Sabbah, A. S, Saad, A. A and Aki, M. Y (to be published )) and the solutions are as follows:-

$$n_+ = 1 - \frac{k}{r} - \frac{\Omega_+^2 k}{2r}\cos^2\theta \tag{6}$$

$$n_- = 1 - \frac{k}{r} - \frac{\Omega_-^2 k}{2r}\cos^2\theta \tag{7}$$

$$\Psi = \frac{b_o}{r} - \frac{\Omega^2 k}{2r}\cos^2\theta \tag{8}$$

where:-

$$k = \frac{\tau(I_{+o} + I_{-o})}{4\pi(1+\tau)}$$

$$b_o = \frac{I_{-o} - \tau I_{+o}}{4\pi(1+\tau)}$$

$$\Omega^2 = \frac{\Omega_+^2 + \Omega_-^2}{2}$$

equations (4) can be separated for ions and electrons to give:

$$\nabla^2 n_+ + \tau \vec{\nabla}.\left(n_+ \vec{\nabla}\Psi\right) + \Omega_+^2 \frac{\partial^2 n_+}{\partial z^2} +$$

$$+\tau\Omega_+^2 \frac{\partial}{\partial z}\left(n_+ \frac{\partial \Psi}{\partial z}\right) + \tau\Omega_+^2\left(\frac{\partial n_+}{\partial x}\frac{\partial \Psi}{\partial y} - \frac{\partial n_+}{\partial y}\frac{\partial \Psi}{\partial x}\right) = 0$$

$$\nabla^2 n_- - \vec{\nabla}.\left(n_- \vec{\nabla}\Psi\right) + \Omega_-^2 \frac{\partial^2 n_-}{\partial z^2}$$

$$-\Omega_-^2 \frac{\partial}{\partial z}\left(n_- \frac{\partial \Psi}{\partial z}\right) - \Omega_-^2\left(\frac{\partial n_-}{\partial x}\frac{\partial \Psi}{\partial y} - \frac{\partial n_-}{\partial y}\frac{\partial \Psi}{\partial x}\right) = 0$$

Using spherical coordinates and dropping the dependence on the angle $(\phi)$ due to symmetry :

$$\nabla^2 n_+ + \tau \vec{\nabla}.\left(n_+ \vec{\nabla}\Psi\right)$$

$$+\Omega_+^2 \left[\cos^2\theta \frac{\partial^2 n_+}{\partial r^2} - \frac{2\sin\theta\cos\theta}{r}\frac{\partial^2 n_+}{\partial r\partial\theta} + \right.$$

$$\left. + \frac{\sin^2\theta}{r^2}\frac{\partial^2 n_+}{\partial\theta^2} + \frac{\sin^2\theta}{r}\frac{\partial n_+}{\partial r} + \frac{2\sin\theta\cos\theta}{r^2}\frac{\partial n_+}{\partial\theta}\right] = 0 \qquad (9)$$

$$\nabla^2 n_- + \vec{\nabla}.\left(n_- \vec{\nabla}\Psi\right)$$

$$+\Omega_-^2 \left[\cos^2\theta \frac{\partial^2 n_-}{\partial r^2} - \frac{2\sin\theta\cos\theta}{r}\frac{\partial^2 n_-}{\partial r\partial\theta} + \right.$$

$$\left. + \frac{\sin^2\theta}{r^2}\frac{\partial^2 n_-}{\partial\theta^2} + \frac{\sin^2\theta}{r}\frac{\partial n_-}{\partial r} + \frac{2\sin\theta\cos\theta}{r^2}\frac{\partial n_-}{\partial\theta}\right] = 0 \qquad (10)$$

and ,

$$\nabla^2 \Psi = \rho_P^2 \left(n_- - n_+\right) \tag{11}$$

now changing the variable (r) to the variable ( $\zeta$ ) for simplifying the forms of the differential equations where:-

$$r = \frac{1}{\zeta}$$

and introducing a stretched variable (x) suitable for the sheath region :

$$x = c\rho_P^a (d - \zeta) \tag{12}$$

where :-

(c), (a) and (d) are constants (Abdel Aziz. M. A, (1987)). Since $\theta$ disappears, then equations (9) and (10) become:-

$$\nabla^2 n_+ + \tau \vec{\nabla} . \left( n_+ \vec{\nabla} \Psi \right) = 0$$

$$\nabla^2 n_- + \vec{\nabla} . \left( n_- \vec{\nabla} \Psi \right) = 0$$

integrating these equations

$$\vec{J}_+ = -\frac{1}{\tau} \vec{\nabla} n_+ - n_+ \vec{\nabla} \Psi \tag{13}$$

$$\vec{J}_- = -\vec{\nabla} n_- - n_- \vec{\nabla} \Psi \tag{14}$$

substituting equation (12) in equations (11), (13) and (14) and using a stretched variable (N) such that :-

$$n = N \rho_P^{2a-2}$$

$$\frac{I_+}{4\pi} = \frac{c}{\tau} N'_+ - cN_+F \qquad (15)$$

$$\frac{I_-}{4\pi} = \frac{c}{\tau} N'_- - cN_-F \qquad (16)$$

$$d^4c^2F' = N_+ - N_- \qquad (17)$$

where:-

$$F = \frac{-d\psi}{dx}$$

Equations (15), (16) and (17) give also the deifferential equation of the electric field strength in the following form:-

$$F''' + (1+\tau)FF' - \frac{\tau}{2}F^3 - xF - A = 0 \qquad (18)$$

the boundary conditions at the probe surface are :-

$$x = 0, \ F = F_P, \ F' = 0$$

introducting the variable :

$$V = \frac{F}{F_P}, \quad F_P << 1$$

expanding equation (18) and taking the first approximation, the following equation can be determined:-

$$V'' - XV = R \qquad (19)$$

where:-

$$R = \frac{A}{F_P}$$

$$V(x) = c_1 A_i(x) + C_2 B_i(x) + B_i(x) R\pi \int_0^x A_i(t)dt - A_i(x) R\pi \int_0^x B_i(t)dt \qquad (20)$$

where :-

$(C_1)$ and $(C_2)$ are constants

$A_i(x)$ and $B_i(x)$ are the airy functions hich are given by:-

$$A_i(x) = \frac{\sqrt{x}}{3}\left[I_{\frac{1}{3}}\left(\frac{2}{3}x^{\frac{3}{2}}\right) - I_{\frac{-1}{3}}\left(\frac{2}{3}x^{\frac{3}{2}}\right)\right] - \frac{1}{\pi}\sqrt{\frac{x}{3}}\,K_{\frac{1}{3}}\left(\frac{2}{3}x^{\frac{3}{2}}\right)$$

$$B_i(x) = \sqrt{\frac{x}{3}}\left[I_{\frac{1}{3}}\left(\frac{2}{3}x^{\frac{3}{2}}\right) + I_{\frac{-1}{3}}\left(\frac{2}{3}x^{\frac{3}{2}}\right)\right]$$

$$C_1 = \frac{3\Gamma\left(\frac{2}{3}\right)}{2(3)^{\frac{1}{3}}}, \quad C_2 = \frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)}{2(3)^{\frac{1}{3}}}$$ ( Abdel Aziz M. A. (1987)).

$$\therefore \Psi(x) = \Psi_P - \int_0^x F(t)dt$$

$$\Psi(x) = \Psi_p + \frac{\pi A}{\sqrt{3}}\left[\int_0^x A_i(t)dt + \sqrt{3}\int_0^x G_i(t)dt\right]$$

where $x \to \infty$

$$\Psi(x) \cong \Psi_p + \left[\frac{\pi A}{3\sqrt{3}} + \frac{A}{3}(\ell n\, 3 + 2\gamma)\right] + A\,\ell n\, x \qquad (21)$$

comparing equation (21) with the form

$$\Psi(x) = \Psi_p + \sum(\tau, x_p) + A\,\ell n\,x$$

from (Abdel Aziz M. A. (1985)), the segma function is

$$\sum(\tau, x_p) = \frac{\pi A}{3\sqrt{3}} + \frac{A}{3}(\ell n\,3 + 2\gamma) \qquad (22)$$

$$\gamma = 0.5772$$

Conclusion:-

Mathematical closed forms are obtained in this paper for the charge concentration and potential in the quasineutral region and in the electric sheath region. It is clear that there is no effect of the magnetic field in the electric sheath region for the first approxmation. From the final result, the charge concentration and the electron temperature in the main bulk of plasma can be obtained by drawing a graph representing the relation between the potential of the electrostatic probe with respect to the plasma and the current drained. ( Characteristic of the probe ).