# IMPLEMENTATION OF NORA PROGRAM FOR HEAT CONDUCTION WITH MELTING AND FREEZING IN PC-COMPUTER

#### BY

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#### ABSTRACT

NORA program is concerned with a numerical analysis of heat transfer with phase change (e.g melting and freezing) for different initial and boundary conditions. It was installed in PC-Computer. The frozen layer thickness as function of the solidification time are presented in dimensionless form. The accuracy of the data obtained were compared with those taken from the mainframe computer and other available solutions for uni-dimensional geometries and for different phase change materials and were found to be in good agreement.

## INTRODUCTION

Heat conduction in systems undergoing phase transformation is encountered in numerous engineering systems. Examples are, melting, freezing, latent heat thermal storage, welding and casting.

NORA program solve numerically the heat transfer problem with phase change in one and two dimensions for homogeneous and and non-homogeneous materials. The temperature distribution is defined by the initial and boundary conditions and by the moving change heat source. A standard difference scheme is used to solve the time dependent heat transfer problem with given source.

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The equation of any continues mixture of time explicit and imlicit scheme are solved with the alternation direction implicit (ADI) method [1]. At any time step, an iteration procedure is employed to make the temperature field consistent with the moving phase change heat source.

The PC-Computer with Intel 80286 processor and 80287/6 numeric co- processor 20 MB main memory and 1.2 RAM disk was used. The operating system of the PC-Computer is PC-DOS version 2.1. In NORA program 1100 statements (Fortran-4) were fed into the Computer through the keyboard and some modifications were made to let the program run on the PC-Computer.

## 2- Mathematical formulation

The governing heat conduction equation with internal source can be written as [2]:

$$\mathcal{S}_{p}(\partial T/\partial t) + \mathcal{S}_{p} L \mathcal{S}_{(T-T_{F})}(\partial T/\partial t) = \text{div}(\lambda \text{grad } T) \quad (1)$$

where  $\boldsymbol{S}$ ,  $C_p$  and  $\boldsymbol{\lambda}$  are the density, the specific heat and the thermal conductivity respectively, while t is the time, L is the latent heat and  $T_F$  is the fusion temperature. The term  $[\boldsymbol{S} L \, \boldsymbol{\delta} (T-T_F)]$  in equation (1) is the source term and it is equal to the heat absorbed or released per unit volume at the phase boundary. For regions removed away from the phase boundary  $\boldsymbol{\delta} (T-T_F) = 0$ , where  $\boldsymbol{\delta}$  is the delta-function given by:

$$(\delta(T - T_F) = 1 \text{ if } T = T_F \text{ and } \delta(T - T_F) = 0 \text{ if } T \neq T_F).$$

2.1 The initial and boundary conditions
Figure (1) shows the temperature distribution of the region
of interest during solidification in the uni-dimensional case.

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The initial temperature of the phase change material (PCM) which undergoing phase change (Liquid to solid) was selected at the fusion temperature  $T_F$ . The interface between the solid and liquid phases is given as f(x,t) at any given time. The initial condition can be written as follows:

$$T(x,0) = T_{in} \qquad (2)$$

where  $T_{\mbox{in}}$  is the initial temperature, and the boundary condition can be written as follow :

$$(2-IB)IB (\lambda/h)(\partial T/\partial x)|_{x=0} + (1-IB)\lambda(\partial T/\partial x)|_{x=0} + IB T|_{x=0} = 1$$

In equation (3) the three types of boundary conditions, namely the constant wall boundary condition ( $T_w = \text{const.}$ ) the constant thermal flux ( $\dot{q}'' = \text{constant}$ ) and the convective heat transfer boundary condition (h = const., where h is the heat transfer coefficient) are writing together in one equation as they built in NORA program. By using the trigger IB (equal 0 or 1) and using the suitable boundary value B we get:

a) For the first type boundary condition ( $T_w = const.$ ): IB = 1 , h =  $\infty$  and  $T_w = B$  (4)

b) For the second type boundary condition ( $\dot{q}'' = const.$ ):

IB = 0 and 
$$\lambda(\partial T/\partial x)$$
  $_{x=0}$  =  $\dot{q}$ " = B (5)

c) For the third boundary condition:

$$IB = 1$$
 and  $T_W = B$  (6)

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## RESULTS AND DISCUSSION

For the uni-dimensional case the comparison were made between the data of the numerical solution obtained from the PC-Computer, the same obtained from the mainframe, the exact solution by Neumann [3] for the constant wall boundary condition and finally the approximate solution by Megerlin [4] for the other two types boundary condition.

Following Mergerlin [4], and to make the comparison possible and the results more valuable some dimensionless parameters should be defined. The dimensionless frozen layer thickness:

 $(\xi^* - 1) = (\xi^- x_0) / x_0$ , see fig.(1), the dimensionless temperature :

 $\Theta = (\ T_F - T\ )\ /\ (T_F - T_w)\ , \ \text{the dimensionless time}\ = \alpha t/(x_o^2),$  where a is the thermal diffusifity (  $d = \lambda/\ C_p ) \ \text{of the PCM}.$  An important parameter for the phase change problem is the dimensionless phase change number Ph = L/ [C\_p(T\_F - T\_w)], it is the ratio between the latent heat and the sensible heat of the PCM. also the dimensionless thermal flux Q\* = (q'' x\_o)/[\lambda(T\_F - T\_w)], and the Biot number

Bi = (h  $_{\text{O}}$  /  $\lambda_{\text{PCM}}$ ) are significant for this problem.

Figures (2), (3) and (4) show the frozen layer thickness ( $\xi$  - 1) as function of the dimensionless time  $\xi$  for different boundary conditions. The solidified layer thickness ( $\xi$ -  $x_o$ ) was allowed to vary up to 0.09 m. In the present work the phase change materials were selected for example as water ( $T_F$ = 0°C) and steel ( $T_F$ = 1520°C)), it can be any other PCM with known thermophysical properties,  $\xi$ - $C_p$ ,  $\lambda$ , L.

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In figure (2) the data obtained from the PC-Computer and those calculated in the mainframe are nearly the same during the solidification (PCM is steel) of a semi-infinite region bounded by different constant wall temperatures (i.e different Ph numbers). While both data from the PC-Computer and the mainframe are in good agreement with Neumann [3] exact solution for the same conditions.

For the boundary condition constant thermal flux fig. (3) represents ( $\xi^*$  - 1) as function of  $\tau$  for a constant value of  $Q^*(\dot{q}"=15\text{x}10^4~\text{W/m}^2)$ . The results shown in this figure are during the solidification of a semi-infinite region outside cylinder (PCM is steel). The data obtained from the PC-Computer and those are taken from the mainframe are nearly identical and both of them are in good agreement with Megerlin [4] at the same conditions.

Figure (4) shows the same comparison for solidification of water using the convective boundary condition . The wall temperature is taken to be equal to the ambient temperature. The comparison for two Biot numbers (Bi = 0.5 and 1.0) at Ph = 5.4 shows good agreement between the numerical solutions (both data from PC-Computer and mainframe) and Megerlin approximate solution [4] at the same conditions. The maximum deviation being less than 3%.

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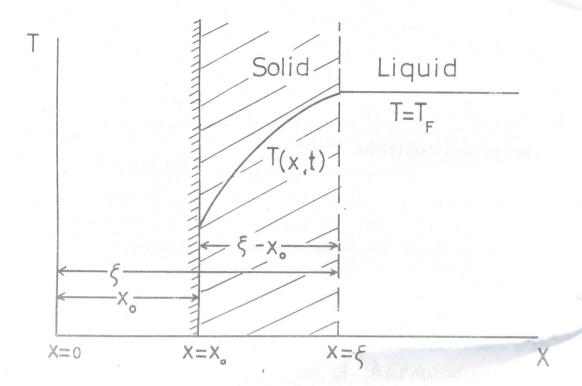


Fig. (1): Temperature distribution in the region of interest during solidification in the uni-dimensional case.

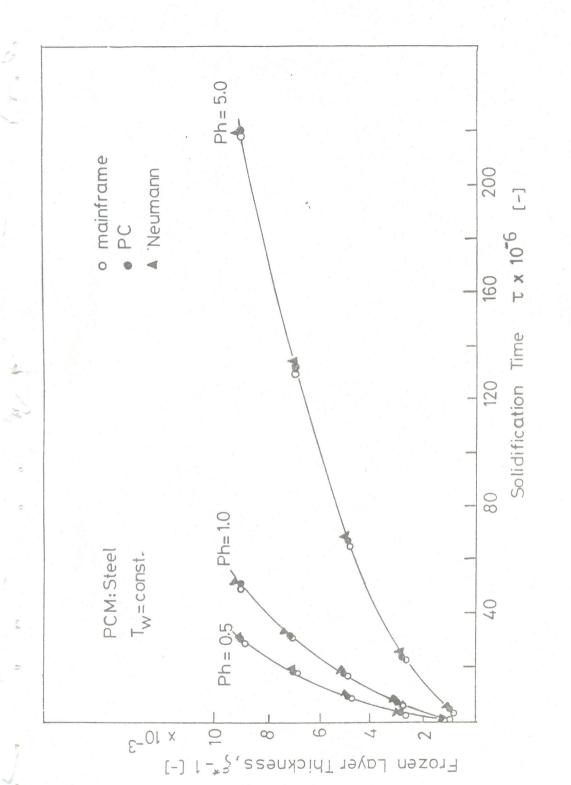


Fig. (2): The frozen layer thickness as function of time during the solidification of steel using the constant wall boundary condition

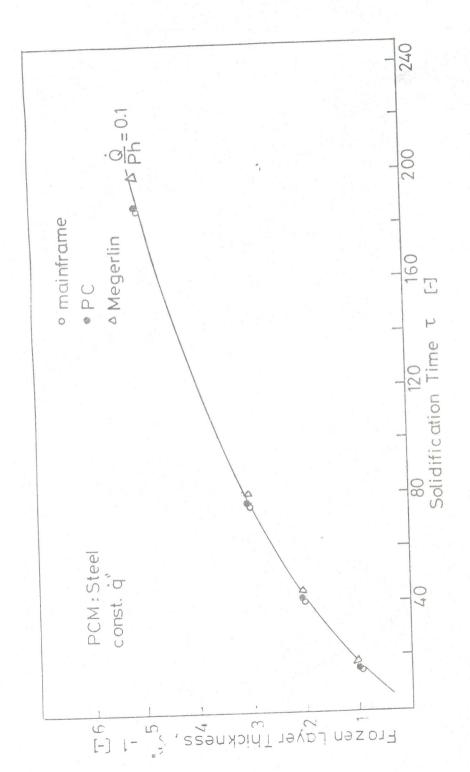


Fig. (3): The frozen layer thickness as function time during solidification of steel using the constant thermal flux boundary condition.

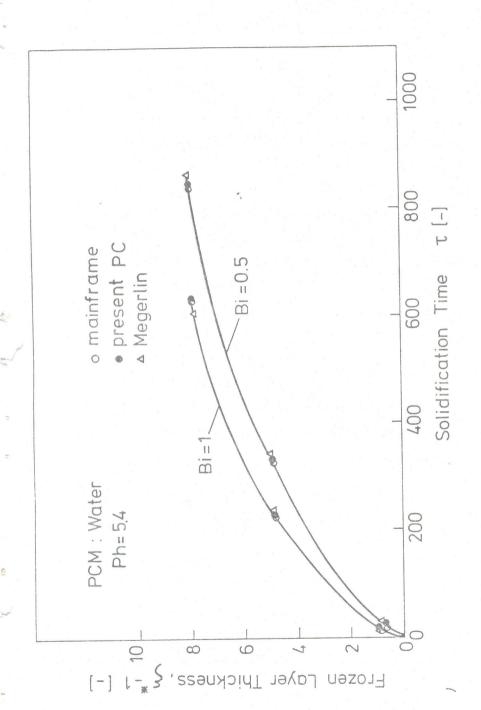


Fig.(4): The frozen layer thickness as function of time during solidification of water for the convective heat transfer boundary.

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## تخرين وتنفيذ برنامج لدراسة التوصيل الحرارى اثناء الانصهار والتجمد على على الحاسب الالى الشخصي

## س\_عد ابو العيــنين

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تم تخرين وتنفيذ برنامج NORA على الحاسب الشخصى والبرنامج يدرس التوصيل الحرارى مع تغرير الحالة (على سبيل المثال اثناء التجمد والانصهار) بشروط ابتدائية وعند ظروف مختلفة بطريقة التحليل العددى وتمت مقارنة سمك الطبقة المتجرمدة كدالة في زمن التجمد التي تم الحصول عليها من الحاسب الشخصى والحاسب المركزى وبعض الحلول الاخرى الموجودة اثناء التجمد في الحالة ذات البعد الواحد وقد وجدت النتائج متطابقة تقريبا والحالة الماكنة الماكنة وقد وجدت النتائج