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S*-CONNECTEDNESS IN BITOPOLOGY

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ABSTRACT

The aim of this paper is to study another type of connectedness in bitopological spaces of a stronger form.

Throughout the paper X-A denotes the complement of A in X and $i,j=1,2$ such that $i \neq j$.

INTRODUCTION

In a topological space X, a set A is semiopen if for some ope set O, $O \subset A \subset \mathrm{cl}O$ where clO denotes the closure of O in X [3] . Complement of a semiopen set is semiclosed. The intersection of all the semiclosed sets containing a set A is called the semiclosure of A and denote it by scl A [1].

The theory of bitopological spaces was first developed by Kelly in 1963 [2]. A bitopological space (X,T_1,T_2) is a non empty set X equipped with two topologies T_1 and T_2. A subset A of a bitopological space (X,T_1,T_2) is turned (i,j) semiopen if there exists a T_i open set O such that $O \subset A \subset T_j\text{-cl}O$, where T_j-clO denotes the closure of O with respect to the topology T_j. Each T_i-open set is (i,j)-semiopen but the converse may be false [2]. Any union of (i,j) - semi open sets is (i,j)- semi open. Complement of an (i,j)- semiopen set is (i,j) semiclosed [2

The intersection of all the (i,j) semiclosed sets containing a set A is turned the (i,j) semiclosure of A. Denote it by (i,j) -scl A. It is the smallest (i,j) - semiclosed set containing A, Further, $A \subset (i,j)$ Scl A Ti-clA, $A \subset B$ implies (i,j)-scl $A \subset (i,j)$- scl B, (i,j)-scl ((i,j)-scl A) = (i,j)-scl A. Moreover A is (i,j)- semiclosed iff A= (i,j) -scl A and $p \in (i,j)$-scl A iff each (i,j) -semiopen set containing p meets A. The study of connectedness in bitopological spaces has been initiated by Pervin [3].

<u>DEFINITION 1 [2]</u>: A function f: $(X, T_1, T_2) \longrightarrow (X^*, T_1^*, T_2^*)$ is said to be pairwise semi continuous if the inverse image of every T_1^* - open set is (i,j) - semiopen in X.

<u>DEFINITION 2 [5]</u>: A function f: $(X,T_1,T_2) \longrightarrow (X^*,T_1^*, T_2^*)$ is said to be pairwise irresolute if the inverse image of every (i,j) -semiopen in X* is -(i,j) semi open in X.

<u>LEMMA 1 [2]</u> : If V is (i,j) semi open and O is biopen in a bitopological space (X, T_1, T_2), then $V \cap O$ is (i,j) - semiopen.

<u>LEMMA 2 [4]</u> : If $(Y, T_1^*, T_2^*,)$ be a biopen subspace of a bitopological space (X, T_1, T_2) and $B \subset Y$, then (j,i)*-scl (B)=(j,i)-scl $B \cap Y$, where (j,i)*-scl (B) means the (j,i) semiclosure of B with respect to Y.

<u>LEMMA 3 [2]</u> : Let (Y, P_1, P_2) be a subspace of a bitopological space (X,T_1,T_2) . If A is (i,j) semiopen in Y and Y is T_i-open in X then A is (i,j) semiopen in X.

<u>LEMMA 4 [2]</u> : If A is (i,j) semiopen and B is biopen in a bitopological space (X, T_1, T_2) then $A \cap E$ is (i,j) semiopen in B.

2. $\langle i,j\rangle$ *-SEMI SEPAPATED SETS

DEFINITION 3 [6] : Two sets A,B in a bitopological space (X,T_1,T_2) are said to be $\langle i,j\rangle$ *-semi separated if $A \cap (i.j)$-scl $(B)=\emptyset=$ (j,i)-scl $(A)\cap B$.

REMARK 1: Any two $\langle i,j)$ *-semiseparated sets are necessarily disjoint. However, the converse may be false, Consider:

EXAMPLE 1: Let $X = \{a,b,c,d\}$, $T_1=\{\emptyset, \{a\}, \{b,c\}, \{a,b,c\}, X\}$, $T_2=\}\emptyset,\{a\}, \{a,b,c\}, X\}$. Then the sets $\{a,b\}$ and $\}c,d\}$ are disjoint but not $\langle i,j\rangle$ *-semi separated.

3. S* - CONNECTED BITOPOLOGICAL SPACES

DEFINITION 4 [6] : A bitopological space (X, T_1, T_2) is s*-connected if it is not the union of two nonempty $\langle i,j\rangle$ *-semi separated sets.

THEOREM 1 : Let a function $f: (X, T_1, T_2) \longrightarrow (X^*,T^*_1, T^*_2)$ be on-to and pairwise semicontinuous. If X s*-connected then X* connected.

THEOREM 2 : Let a function $f: (X,T_1,T_2) \longrightarrow (X^*,T^*_1,T^*_2)$ be onto and pairwise irresolute. If X is s*-connected then X* is s*-connected.

The proofs are straight forward.

LEMMA 5 : Let $f: (X,T_1, T_2) \longrightarrow (X^*, T^*_1 , T^*_2)$ be one-one, pairwise open and pairwise continuous. If B is (i,j)-semiopen in X* then $f^{-1}(B)$ is (i,j)-semiopen in X.

PROOF : Since B is (i,j)-semiopen in X* there is a T^*_1-open set 0 such that $0\subset B\subset T^*_j$-cl (o). Therefore, $f^{-1}(0)\subset f^{-1}(B)\subset f^{-1}(T^*_j\text{-cl}(0))$. Since f is pairwise continuous, $f^{-1}(0)$ is T_i-open. Let $x \in f^{-1}(T^*_j\text{-cl}(0))$ and U be a T_j-open set containing x

Then, $f(x) \in f(U)$. Now $f(U)$ is T^*_j-open for f is pairwise open. Since $f(x) \in T^*_j\text{-cl}(0)$, it follows that $f\text{-}(U) \cap 0 \neq \emptyset$. Now f being one one this implies that $U \cap f^{-1}(0) \neq \emptyset$. And so, $x \in T_j\text{-cl}\,(f^{-1}(0))$. That, $f^{-1}(0) \subset f^{-1}(B) \subset f^{-1}(T^*_j\text{-cl}(0)) \subset T_j\text{-cl}\,(f^{-1}(0))$. Hence, $f^{-1}(B)$ is (i,j)-semiopen in X.

<u>THEOREM 3</u>: If f: $(X,T_1,T_2) \longrightarrow (X^*,T^*_1,T^*_2)$ be a pairwise homeomorphism and X is s*-connected then X* is s*-connected. The proof of the theorem is obtained by using Lemma 5.

<u>DEFINITION 5</u>: A bitopological space (X, T_1, T_2) is locally s*-connected if for every point $x \in X$ and every T_i-open set 0 containing x there exists a biopen s*-connected set G such that $x \in G \subset 0$.

<u>REMARK 2:</u> A locally s*-connected space need not be s*-connected. Consider $X = \{a,b\}$ and $T_1 = T_2 = \{\emptyset, \{a\}, \{b\}, X\}$.

<u>REMARK 3</u> : A space may be s* -connected without being locally s*-connected. Consider: $X = \{a,b\}$, $T_1 = \{\emptyset, \{a\}, \{b\}, X\}$ and $T_2 = \{\emptyset, X\}$.

<u>THEOREM 4</u> : Every biopen subspace (F,T^*_1,T^*_2) of a locally s*-connected space (x,T_1,T_2) is locally s*-connected.

<u>DEFINITION 6</u>: Let X be a locally s*-connected space and $p \in X$, the s*-component of p is the union of all the biopen s*-connected sets containing the point p.

<u>THEOREM 5</u> : Each s*-component is biopen and s*-connected.

<u>THEOREM 6</u> ; Each s*-component of a biopen set of a locally s*-connected space is biopen.

This follows from theorem 4 and 5.

THEOREM 7 ; Two distinct s*-components are <i,j> *-semiseparated.

COROLLARY 1 : The family of all the s*-components, of a locally-s*-connected space X, is a partition of X.

The proofs are straight forward.

THEOREM 8 : Each s*-components is biclosed.

This follows from theorem 5 and Corollary 1.

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الترابط فى ثنائى التوبولوجى

الهدف من البحث هو دراسة نوع اخر من انواع الترابط
ذات الشكل الاقوى فى الفراغات ثنائية التوبولوجى.
خلال البحث سنكتب $A - X$ لنعنى بذلك مكمل المجموعة A
فى الفراغ X كما ان $i,j = 1,2$ حيث $i = j$