

SOME MORE RESULTS ON FUZZY CONTINUITY

BY

A.A. Allam

Mathematics Department, Faculty of Science, Assiut University

Assiut, Egypt

Received: 27-5-1992

ABSTRACT

In this peice of work we define α^* -semicontinuous, $R\alpha$ - semicontinuous and $R\alpha^*$ -semicontinuous functions as a generalization of F-semicontinuous and F-irresolute functions. By these concepts and the concept of an α -continuous functions, we improve and generalize (in fuzzy setting) some results previously obtained by Malghan and Benchalli [11,12] and Sivaraj [17].

1. INTRODUCTION

In 1978, Gantner, et al. [9] introduced the concept of α -compact and strong locally α -compact fuzzy spaces. In 1979 Gameron [8] introduced for the first the concept of I-compact spaces. In 1980 and 1984, the concept of countably α -compact and locally α -compact fuzzy spaces were introduced by Malghan and Benchalli [11,12]. In 1986 and 1987 the concepts of α -continuity α -irresolute functions and S-closed spaces were introduced by Mashhour et al. [13,15].

Delta J. Sci. 16 (1) 1992

Some More Results

Theorem A [17]. If a function $f: X \rightarrow Y$ is irresolute and $A \subset X$ is I-compact relative to X , then $f(A)$ is α S-closed relative to Y .

Corollary B [17]. If a function $f: X \rightarrow Y$ is irresolute, Y is an FED-space and $A \subset X$ is I-compact relative to X , then $f(A)$ is I-compact relative to Y .

Corollary C [17]. If a function $f: X \rightarrow Y$ is semiopen and semicontinuous, Y is an ED-space and $A \subset X$ is I-compact relative to X , then $f(A)$ is I-compact relative to Y .

Theorem D [11]. Let $f: X \rightarrow Y$ be an F-continuous function. If X is countably α -compact, then $f(X)$ is countably α -compact subspace of Y .

Theorem E [11]. Let $f: X \rightarrow Y$ be an F-continuous, F-open surjection. If X is locally α -compact, then Y is locally α -compact.

Corollary F [12]. Let f be an F-open, F-continuous function from an fts X onto a 1^* -Hausdorff fts Y such that $\text{Supp}(\text{Cl}(A)) = \text{Cl}(\text{Supp}(A))$ for each fuzzy open set A in Y . If X is strong locally α -compact, then so is Y .

Let S be a fuzzy subset of a fuzzy topological space (an fts, for short) (X, τ) , we denote the closure of (resp. interior of S , semiclosure of S) with respect to τ by $\text{Cl}(S)$ (resp. $\text{Int}(S)$, $\text{ScI}(S)$). The family of all fuzzy semiopen (resp. fuzzy regular semiopen) sets in X are denoted by $\text{SO}(X)$ (resp. $\text{RSO}(X)$).

Delta J. Sci. 16 (1) 1992

A.A. Allam

Definition 1.1 . A fuzzy subset S of an fts X is called:

- (i) fuzzy semiopen [2,5] if $S \leq Cl(Int(S))$,
- (ii) fuzzy semiclosed [2,5] if $S \geq Int(Cl(S))$,
- (iii) fuzzy preopen [7] if $S \leq Int(Cl(S))$,
- (iv) fuzzy regular open [5] if $S = Int(Cl(S))$,
- (v) fuzzy regular-semiopen [13] if there is a fuzzy regular open set U such that $U \leq S \leq Cl(U)$.

Definition 1.2 [16]. A fuzzy point x_r is said to be quasicoincident with A , denoted by $x_r qA$ if $r > A''(x) > 1$. A fuzzy set A is quasi-coincident with a fuzzy set B or $A qB$ if there is $x \in X$, $A(x) + B(x) > 1$. A fuzzy set A in an fts (X, τ) is called a Q -neighbourhood of x_r iff there is $B \in \tau$ such that $x_r qB \leq A$.

Theorem 1.3. [16]. A fuzzy point $x_r \in Cl(A)$ iff each q -neighbourhood of x_r is quasi-coincident with A .

The following Propositions have been stated in [10].

Proposition 1.4. (i) If $A \cap B = 0$, then $A qB$ (A is not quasi-coincident with B),

(ii) $A \leq B$ iff $x_r qB$ for each $x_r qA$,

(iii) $x_r qA$ iff $x_r \in A''$ (A'' is the complement of A),

(iv) $A qB$ iff $A \leq B''$,

(v) $x_r q(\bigvee_j A_j)$ iff there is $j_0 \in J$ such that $x_r qA_{j_0}$

Delta J. Sci. 16 (1) 1992

Some More Results

Proposition 1.5. Let (X, τ) be an fts and $V \in \tau$, then $\forall qA \iff \forall qCl(A)$ for each fuzzy set A in X .

Theorem 1.6. [14]. A fuzzy point $x_r \in Scl(A)$ iff each fuzzy semiopen set U with $x_r qU$, UqA .

Theorem 1.7 [12]. If $f: X \rightarrow Y$ is a function from a set X into a set Y , A is a fuzzy subset in X and B is a fuzzy subset in Y , then

- (i) $f(S(A)) = S(f(A))$, where $S(A)$ means support of A .
- (ii) $f^{-1}(S(B)) = S(f^{-1}(B))$.

Definition 1.8. Let $f: X \rightarrow Y$ be a function from an fts X into an fts Y . Then f is called:

- (i) F -continuous [6] if $f^{-1}(V)$ is fuzzy open in X for each fuzzy open set in Y ,
- (ii) F -open [6], if $f(U)$ is fuzzy open in Y for each fuzzy open set U in X .
- (iii) F -irresolute [14] if $f^{-1}(V) \in SO(X)$ for each $V \in SO(Y)$.
- (iv) F -semicontinuous [2,5] if $f^{-1}(V) \in SO(X)$ for each fuzzy open set V in Y .
- (v) α -continuous [13], if for each $x \in X$ and for each fuzzy open set V in Y with $V(f(x)) > \alpha$, there is a fuzzy open set U in X with $U(x) > \alpha$ and $f(U) \leq V$, $0 < \alpha < i$.
- (vi) α -semicontinuous (resp. α -irresolute) [15], if for $x \in X$ and for each fuzzy open set V in Y (resp. $V \in SO(Y)$) with $V(f(x)) > \alpha$, there is $U \in SO(X)$ with $U(x) > \alpha$ and $f(U) \leq V$.

Delta J. Sci. 16 (1) 1992

A.A. Allam

Definition 1.9. [9]. Let X be an fts and $\alpha \in [0,1]$. A collection \mathcal{V} of fuzzy sets is called α -shading of X if for each $x \in X$, there is $U \in \mathcal{V}$ with $U(x) > \alpha$. A subcollection of an α -shading \mathcal{V} which is also an α -shading is called an α -subshading of \mathcal{V} .

Definition 1.10. An fts X is called;

- (i) Countably α -compact [11], if every countable open α -shading of X has a finite α -subshading.
- (ii) α -compact [9] (resp. α -almost compact [1]) if every open α -shading of X has a finite α -subshading (resp. has a finite subfamily the closure of whose members are an α -shading of X).
- (iii) Locally α -compact [12] (resp. strong locally α -compact [9]) if for each $x \in X$, there is a neighbourhood U of x (resp. an open neighbourhood) of x such that $U(x) = 1$ and $S(U)$ (resp. $S(Cl(U))$) is α -compact.
- (iv) α S-closed [13] (resp. α I-compact [4]) if for each semiopen α -shading of X , there is a finite subfamily the closure (resp. the interior of the closure) of whose members are an α -shading of X .

Definition 1.11. A fuzzy subset A of an fts X is called:

- (i) α S-closed [3] (resp. α I-compact [4]) relative to X if for each α -shading of $S(A)$ by fuzzy semiopen sets of X ; there is a finite subfamily the closure (resp. the

Bolton J. Sci. 16 (1) 1992

Some More Results

interior of the closure) of whose members are an α -shading of $S(A)$.

(ii) α -almost compact relative to X if for each α -shading of $S(A)$ by fuzzy open set of X , there is a finite subfamily the closure of whose members are an shading of $S(A)$.

Definition 1.12. [13]. An fts X is called fuzzy extremely disconnected space (an FED-space, for-short) if the closure of each fuzzy open set in X is fuzzy open in X .

2. α^* -semicontinuity $R\alpha^*$ -semicontinuity and αI -compactness.

Definition 2.1. Let $f:(X, \tau) \rightarrow (Y, \theta)$ be a function from an fts (X, τ) to an fts (Y, θ) . For $0 \leq \alpha < 1$, f is called:

(i) α^* -semicontinuous if for each $x \in X$ and for each $V \in \theta$, with $V(f(x)) > \alpha$, there is $U \in SO(X)$ such that $U(x) > \alpha$ and $f(\text{Int}(\text{Cl}(U))) \leq 1(V)$.

(ii) $R\alpha$ -semicontinuous (resp. $R\alpha^*$ -semicontinuous) if for each $x \in X$ and for each $V \in RSO(Y)$ with $V(f(x)) > \alpha$, there is $U \in SO(X)$ such that $U(x) > \alpha$ and $f(U) \leq V$ (resp. $f(\text{Int}(\text{Scl}(U))) \leq \text{Scl}(V)$).

Remark 2.2. It is clear that an F -semicontinuous (resp. F -irresolute) function is an α^* -semicontinuous (resp. $R\alpha$ -semicontinuous and $R\alpha^*$ -semicontinuous) function. That the converses need not be true as shown by Examples 2.3 and 2.4, below.

Example 2.3. Let $X = \{a, b\}$, U_1 , U_2 and U_3 be fuzzy subsets of X defined as

Delta J. Sci. 16 (1) 1992

A.A. Allam

$$\begin{array}{ll} U_1(a) = 0.6 & U_1(b) = 0.55, \\ U_2(a) = 0.4 & U_2(b) = 0.30, \\ U_3(a) = 0.7 & U_3(b) = 0.8. \end{array}$$

Consider the following fuzzy topologies $\tau = \{0, U_1, 1\}$ & $\theta = \{0, U_2, U_3, 1\}$ on X and the function $i: (X, \tau) \rightarrow (X, \theta)$ defined by $i(x) = x$ for each $V_j(a) = r_j \geq 0.6$, $V_j(b) = S_j \geq 0.55$; it is obvious that i is α^* -semicontinuous which is not F -semicontinuous when $0.4 < \alpha < 0.5$.

Example 2.4. Let $X = \{a, b\}$, V_1, V_2 and V_3 be fuzzy subsets of X defined as

$$\begin{array}{ll} V_1(a) = 0.45 & V_1(b) = 0.51 \\ V_2(a) = 0.4 & V_2(b) = 0.45 \\ V_3(a) = 0.6 & V_3(b) = 0.5 \end{array}$$

Consider the fuzzy topologies $\tau = \{0, V_1, 1\}$ and $\theta = \{0, V_2, V_3, 1\}$ on X and the function $i: (X, \tau) \rightarrow (X, \theta)$ defined by $i(x) = x$ for each $x \in X$. It is clear that all fuzzy semiopen sets in τ are U_j where $U_j(a) = r_j > 0.45$, $U_j(b) = S_j \geq 0.51$. Also, the only fuzzy regular semiopen sets in θ are H_j and

M_j where $H_j(a) = 0.4$, $0.45 < H_j(b) < 0.5$, $M_j(a) = 0.6$ and $0.5 < M_j(b) < 0.55$. It is obvious that i is R_α -semicontinuous and R_{α^*} -semicontinuous but not F -irresolute when $\alpha = 0.53$.

Lemma 2.5. Let X be an fts and U be a fuzzy subset of X , then $\text{Int}(\text{Cl}(U)) \leq \text{Scl}(U)$.

Proof: Let $x_r \notin \text{Scl}(U)$, then there is $V \in \text{So}(X)$ with $x_r q V$ such that $V q U$ and hence $\text{Int}(V) q U$. By Proposition 1.5, $\text{Int}(V) q \text{Cl}(U)$ and hence $\text{Cl}(\text{Int}(V)) q \text{Int}(\text{Cl}(U))$. By Proposition 1.4, $\text{Cl}(\text{Int}(V)) < 1 - \text{Int}(\text{Cl}(U))$. Since $V \in \text{So}(X)$ and $x_r q V$, then $x_r q \text{Cl}(\text{Int}(V))$ and hence $x_r q (1 - \text{Int}(\text{Cl}(U)))$. By Proposition 1.4, $x_r \notin \text{Int}(\text{Cl}(U))$. This show that $\text{Int}(\text{Cl}(U)) \leq \text{Scl}(U)$.

Lemma 2.6. Let X be an fts, then $\text{Scl}(U) \leq \text{Int}(\text{Cl}(U))$ for each fuzzy preopen set U of X .

Proof: Let $x_r \notin \text{Int}(\text{Cl}(U))$, then $x_r q (1 - \text{Int}(\text{Cl}(U)))$. Put $1 - \text{Int}(\text{Cl}(U)) = V \in \text{So}(X)$. Then $x_r q V$. Since U is fuzzy preopen,

Delta J. Sci. 16 (1) 1992

A.A. Allam

then $U \subseteq \text{Int}(\text{Cl}(U))$, which implies $U \cap (1 - \text{Int}(\text{Cl}(U))) = \emptyset$.
Hence, there is $V \in \text{RSO}(X)$ with $x \in V$ such that $V \cap U = \emptyset$. Then
 $x \notin \text{Scl}(U)$. This shows that $\text{Scl}(U) \subseteq \text{Int}(\text{Cl}(U))$.

Lemma 2.7. A fuzzy subset U of an fts X is fuzzy regular semiopen if and only if U is fuzzy semiopen and fuzzy semiclosed.

Proof: Let $U \in \text{RSO}(X)$, then there is a fuzzy regular open set V of X such that $V \subseteq U \subseteq \text{Cl}(V)$ and hence $\text{Int}(\text{Cl}(U)) = \text{Int}(\text{Cl}(V)) = V \subseteq U$. Therefore U is fuzzy semiclosed and fuzzy semiopen. Conversely, let U be fuzzy semiopen and fuzzy semiclosed in X , then $\text{Int}(\text{Cl}(U)) \subseteq U \subseteq \text{Cl}(\text{Int}(U)) \subseteq \text{Cl}(\text{Int}(\text{Cl}(U)))$. Put $\text{Int}(\text{Cl}(U)) = V$. Which is fuzzy regular open in X . Hence there exists a fuzzy regular open set V of X such that $V \subseteq U \subseteq \text{Cl}(V)$. Then U is fuzzy regular semiopen.

Theorem 2.8. If (Y, θ) is an FED-space and a function $f: (X, \tau) \rightarrow (Y, \theta)$ is α -semicontinuous, then f is $R\alpha$ -semicontinuous.

Proof: Let $x \in X$ and $V \in \text{RSO}(Y)$ with $V(f(x)) > \alpha$. Since Y is an FED-space, then $V \subseteq \text{Cl}(\text{Int}(V)) = \text{Int}(\text{Cl}(\text{Int}(V)))$, with $\text{Int}(\text{Cl}(\text{Int}(V)))(f(x)) > \alpha$. Since f is α -semicontinuous,

Delta J. Sci. 16 (1) 1992

SOME MORE RESULTS

There is $U \in SO(X)$ such that $U(x) > \alpha$ and $f(U) \leq \text{Int}(\text{Cl}(\text{Int}(V)))$.

By Lemma 2.5 and 2.7, $\text{Int}(\text{Cl}(V)) \leq \text{Sc1}(V) = V$ and hence

$f(U) \leq V$, then f is $R\alpha$ -semicontinuous.

Theorem 2.9. If (Y, θ) is an FED-space and a function $f: (X, \tau) \rightarrow (Y, \theta)$ is α^* -semicontinuous, then f is $R\alpha^*$ -semicontinuous.

Proof: Let $x \in X$ and $V \in RSO(Y)$ with $V(f(x)) > \alpha$. Similar to that of Theorem 2.8., there is $U \in SO(X)$ such that $U(x) > \alpha$ and U is α -semiclosed, then $\text{Int}(\text{Sc1}(U)) = \text{Int}(\text{Cl}(\text{Sc1}(U))) = \text{Int}(\text{Cl}(U))$. Since $V \in RSO(Y)$, then $\text{Cl}(V) = \text{Cl}(\text{Int}(V)) = \text{Int}(\text{Cl}(\text{Int}(V))) \leq \text{Sc1}(V)$. Hence $f(\text{Int}(\text{Sc1}(U))) \leq \text{Sc1}(V)$. Therefore f is $R\alpha^*$ -semicontinuous.

Theorem 2.10. If a function $f: X \rightarrow Y$ is α^* -semiconuuous and A is α I-compact relative to X , then $f(A)$ is α -almost compact relative to Y .

Proof: Let \mathcal{V} be a fuzzy open α -shading of $S(f(A))$. Thus for $y \in S(f(A))$, there is $V_y \in \mathcal{V}$ such that $V_y(y) > \alpha$. Since f is α^* -semicontinuous, then for $x \in S(A)$ with $f(x)=y$, there is $U_x \in SO(X)$ such that $U_x(x) > \alpha$ and $f(\text{Int}(\text{Cl}(U_x))) \leq \text{Cl}(V_y)$.

Delta J. Sci. 16 (1) 1992

A.A. Allan

Hence $\{U_x : x \in S(A)\}$ is an α -shading of $S(A)$ of fuzzy semiopen sets of X . Since A is α I-compact relative to X , then there is a finite subfamily $\{U_{x_i} : i = 1, 2, 3, \dots, n\}$ such that $\{\text{Int}(\text{Cl}(U_{x_i})) : i=1, 2, 3, \dots, n\}$ is an α -shading of $S(A)$.

For $y_i \in S(f(A))$, there is $x_i \in S(A)$ such that $f(x_i) = y_i$

and hence there is $\text{Int}(\text{Cl}(U_{x_i}))$ such that $\text{Int}(\text{Cl}(U_{x_i}))(x_i) > \alpha$.

But $f(\text{Int}(\text{Cl}(U_{x_i}))) (y_i) = \sup_{x_i \in f^{-1}(y_i)} (\text{Int}(\text{Cl}(U_{x_i}))(x_i) > \alpha$,

which implies for $y_i \in S(f(A))$, $\text{Cl}(V_{y_i})(y_i) > \alpha$. Then $\{\text{Cl}(V_{y_i}) : i=1, 2, 3, \dots, n\}$ is an α -shading of $S(f(A))$. Therefore $f(A)$ is α -almost compact relative to Y .

In an fts X , it is easy to see that if A is α I-compact relative to X , then A is α S-closed relative to X , and α -almost compact relative to X , but not the converse. Also by the following Theorem we show that, in an FED-space X if A is α -almost compact relative to X , then A is α I-compact relative to X .

Theorem 2.11. If X is an FED-space and A is α -almost compact relative to X , then A is α I-compact relative to X .

Delta J. Sci. 16 (1) 1992

Some more Results

Proof: Let $\{F_i: i \in I\}$ be an α -shading of $S(A)$ by fuzzy regular closed sets in X . Since X is an FED-space, then for each $i \in I$, $F_i = Cl(Int(F_i))$, which is fuzzy open in X . Since A is α -almost compact relative to X , then there is a finite subset I_0 of I such that $\{Cl(F_i) : i \in I_0\}$ is an α -shading of $S(A)$ since X is an FED-space, then $Int(F_i) = F_i = Cl(F_i)$ and hence $\{Int(F_i): i \in I_0\}$ is an α -shading of $S(A)$. Then A is α I-compact relative to X .

The following corollary may be considered as a generalization (in fuzzy setting) and an improvement of Corollaries B and C.

Corollary 2.12. If a function $f: X \rightarrow Y$ is α^* -semi-continuous, Y is an FED-space and A is α I-compact relative to X , then $f(A)$ is α I-compact relative to Y .

Proof: It follows that from Theorems 2.10 and 2.11. The following Theorem may be considered as a generalization (in fuzzy setting) and an improvement of Theorem A.

Theorem 2.13. If a function $f: X \rightarrow Y$ is R^* -semi-continuous and A is α I-compact relative to X , then $f(A)$ is α S-closed relative to Y .

Delta J. Sci. 16 (1) 1991

A.A. Allam

Proof: Let \mathcal{V} be an α -shading of $S(f(A))$ by fuzzy regular semiopen sets of Y . Thus for $y \in S(f(A))$, there is $V_y \in \mathcal{V}$ such that $V_y(y) > \alpha$. Since f is $R\alpha^*$ -semicontinuous, then for $x \in S(A)$, with $f(x)=y$, there is $U_x \in SO(X)$ such that $U_x(x) > \alpha$ and $f(\text{Int}(\text{Scl}(U_x))) < \text{Scl}(V_y)$. One can obtain as in Theorem 2.11., for $y_i \in S(f(A))$, there is $x_i \in S(A)$ with $f(x_i)=y_i$ and hence, there is $\text{Int}(\text{Cl}(U_{x_i}))$ such that $\text{Int}(\text{Cl}(U_{x_i}))(x_i) > \alpha$. But $\text{Int}(\text{Cl}(U_{x_i})) = \text{Int}(\text{Scl}(U_{x_i}))$, then $f(\text{Int}(\text{Scl}(U_{x_i}))) (y_i) = (\text{Int}(\text{Cl}(U_{x_i}))) (y_i) = \sup_{x_i \in f^{-1}(y_i)} (\text{Int}(\text{Cl}(U_{x_i}))) (x_i) > \alpha$ which implies for $y_i \in S(f(A))$, $\text{Scl}(V_{y_i})(y_i) > \alpha$. Then $\{ \text{Cl}(V_{y_i}) : i=1,2,3, \dots, n \}$ is an α -shading of $S(f(A))$. Therefore $f(A)$ is α S-closed relative to Y .

Corollary 2.14. If a function $f: X \rightarrow Y$ is $R\alpha^*$ -semicontinuous, Y is an FED-space and A is α I-compact relative to X , then $f(A)$ is α I-compact relative to Y .

The following corollary may be considered as a generalization (in fuzzy setting) and an improvement of Corollaries B and C.

Corollary 2.15. If a function $f: X \rightarrow Y$ is α^* -semicontinuous, Y is an FED-space and A is α I-compact relative

to X , then $f(A)$ is α I-compact relative to Y .

Proof: It follows that from Theorem 2.9 and Corollary 2.14.

In 1986, Mashhour, et al. [13], show that every F -continuous function is an α -continuous function, but not the converse. Since it is so, then theorems 2.16, 2.17 and Corollary 2.18 may be considered as an improvement of Theorems D, E and Corollary F, respectively.

Theorem 2.16. If a function $f: X \rightarrow Y$ is α -continuous and X is countably α -compact, then $f(X)$ is countably α -compact.

Proof: Assume that $f(X)=Y$. Let \mathcal{V} be a countable open α -shading of Y . Thus for $y \in Y$, there is $U_y \in \mathcal{V}$ such that $Y_y(y) > \alpha$. Since f is α -continuous, then for $x \in X$ with $f(x)=y$, there is a fuzzy open set V_x of X such that $V_x(x) > \alpha$ and $f(V_x) \leq U_y$. Hence $\mathcal{U} = \{V_x : x \in X\}$ is countable open α -shading of X . Since X is countable α -compact, then \mathcal{U} has a finite α -subshading $\{V_{x_i} : i=1, 2, 3, \dots, n\}$. For $y_i \in Y$, there is $x_i \in X$ with $f(x_i) = y_i$ and hence there is V_{x_i} such that $V_{x_i}(x_i) > \alpha$. But $f(V_{x_i})(y_i) = \sup_{x_i \in f^{-1}(y_i)} V_{x_i}(x_i)$, which $U_{y_i}(y_i) > \alpha$. Hence $\{U_{y_i} : i=1, 2, 3, \dots, n\}$ is a finite α -shading of Y . Then $f(X)$

Delta J. Sci. 16 (1) 1992

A.A. Allam

is countably α -compact.

Theorem 2.17. Let $f: X \rightarrow Y$ be an α -continuous, F-open surjection. If X is locally α -compact, then Y is also locally α -compact.

Proof: Let $y \in Y$ and let $f(x) = y$. Since $x \in X$, there is a neighbourhood U of x such that $U(x)=1$ and $S(U)$ is α -compact. Since f is F-open, then $f(U)$ is a neighbourhood of y such that $(f(U))(y) = \sup_{x \in f^{-1}(y)} U(x) = 1$ and since f is α -continuous $f(S(U)) = S(f(U))$ is α -compact [13 Corollary 4.9]. Hence Y is locally α -compact.

Corollary 2.18. Let f be an α -continuous, F-open function from an fts X onto a 1^* -Hausdorff fts Y such that $S(Cl(U)) = Cl(S(U))$ for each fuzzy open set U in Y . If X is strongly locally α -compact, then so is Y .

Proof: It follows that from Theorem 2.17 and Theorem 5.5 of [12].

Delta J. Sci. 16 (1) 1992

some more results

REFERENCES

- 1- M.E. Abd-El-Monsef and M.H. Ghanim, Almost compact fuzzy topological spaces, Delta J.Science 5(1981), 19-29.
- 2- -----, On semiopen fuzzy sets, Delta J. Science 5 (1981), 30-44.
- 3- A.A.Allam and A.M. Zahran, Functions and α S-closedness on fuzzy topological spaces, Fuzzy sets and Systems (to appear).
- 4- -----, Functions and α I-compactness on fuzzy topological spaces (submitted).
- 5- K.K. Azad, Fuzzy semicontinuity, fuzzy almost continuity and fuzzy weakly continuity, J.Math.Anal.Appl. 42(1981), 14-32.
- 6- C.L. Chang, Fuzzy topological spaces, J.Math. Anal.Appl. 24(1968) 182-190.
- 7- M.A. Fath Alla, On fuzzy topological spaces, Ph.D. Thesis Assiut Univ. (1984).
- 8- D.E. Cameron, Some maximal topologies which are QHC, Proc. Amer. Math. Soc., 75 (1979) 149-156.
- 9- T.F. Ganter, R.C. Steinlage and R.H. Warren, Compactness in fuzzy topological spaces, J.Math. Anal. Appl. 62(1978), 547-562.
- 10- A.Kandil and A.M. El-Etriby, On separation axioms in fuzzy Topological spaces. Tamkang J.Math., 18(1987) 49-59
- 11- S.R. Malghan and S.S. Beachalli, On fuzzy topological spaces Ind. Math. 16(1981), 313-325.

Delta J. Sci. 16 (1) 1992

A.A. Allam

- 12- S.R. Malghan and S.S. Benchalli, Open maps, closed maps and local compactness in fuzzy topological spaces, J. Math Anal. Appl. 9(1984), 338-349.
- 13- A.S. Mashhour, M.H. Ghanim and M.A. Fath Alla, α -separations axioms and α -compactness in fuzzy topological spaces. Rocky Mount. J. Math., 3(1986), 591-600.
- 14- A.S. Mashhour, A.A. Allam and F.M. Zeyads, fuzzy regular closed functions and fuzzy irresolute functions, Bull, Fac. Sci. Assiut Univ., 14(2)(1985), 43-59.
- 15- A.S. Mashhour, A.A. Allam and K.M. Abd El-Hakeim, on fuzzy semicompact spaces, Bull. Fac.Sci., Assiut Univ. 16(1) (1987), 277-285
- 16- Pu. Poe-Ming and Liu Ying-Ming, Fuzzy topology, 1. Neighborhood structure of a fuzzy point and Moore smith convergence, J.Math. Anal. Appl., 76 (1980), 571-599.
- 17- D.Sivaraj, A note on S-closed spaces, Acta, Sci. Hunger 44 (1984), 207-213.

بعض النتائج الاضافية على الاتصال الغازى

احمد عبد النصف علام

قسم الرياضيات كلية العلوم جامعة اسيوط

فى هذا البحث عرفنا مفاهيم شبه الاتصال الغازى من النوع * α وشبه الاتصال الغازى من النوع R_α والنوع R_{α^*} كتعميم لشبه الاتصال الغازى . باستخدام تلك المفاهيم ومفهوم الاتصال من النوع α تم تحسين وتعميم بعض النتائج فى [11,12,17]