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MINKOWSKI SPACE AND ITS FOLDING PLANES

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ABSTRACT

In this paper we proved that the folding planes of any isometric folding of Minkowski space into itself keeping the origin fixed are transverses to the null cone.

1- INTRODUCTION

A map $f : M \longrightarrow N$, where M and N are C^{∞} Riemannian manifolds of dimensions m and n respectively, is said to be isometric folding of M into N iff for any piecewise geodesic path $\gamma : J \longrightarrow M$, the induced path $f_{o}\gamma : J \longrightarrow N$ is piecewise geodesic and of the same length [2].

The above definition makes no essential use of the fact that the Riemannian structure g is positive definite. All it is required is that geodesic segments are well-defined and can be assigned "length". This suggest that the concept of isometric folding can be extended to pseudoriemannian manifolds [1].

Let $f : M \longrightarrow N$ be an isometric folding, the set of all points $x \in M$ at which f is not differentiable is called the set of all singularities of f [2]. This set is the union of what we shall call <u>k-folding planes</u> $o \leqq k \leqq m-1$, where a 0-folding plane is a point, a 1-folding plane is aline and a 2-folding plane is a plane and so on.

Now, suppose that $f : M^4 \longrightarrow M^4$ is an isometric folding of Minkowski M^4 space into itself such that $f(0) = 0$, $f(x) = x$, $x \neq 0$. Then $f(tx) = tx$, for all $t \in R$. Suppose further that $x \in$ null cone, that is $\| x \| = 0$. Then the expecting folding planes will be of dimensions k where $o \leqq k \leqq 3$. These folding planes have the property that $f(y) = y$ for all y in the folding planes. The following two theorems will grantee that the 2-folding planes will be transverse to the null cone.

2- Theorem 1

Let L be a time-sense preserving Lorentz 4×4 matrix with $dt\ L = 1$. If 1 is an eigenvalue of L, then the dimension of its eigenspace is either 2 or 4.

Proof. The time-sense preserving Lorentz 4×4 matrix has the form $\begin{bmatrix} O_3 & 0 \\ 0 & 1 \end{bmatrix}$, where $O_3 = [a_{ij}]$ is a 3-dimensional orthogonal matrix.

Let $\lambda_1, \lambda_2, \lambda_3$ and 1 be the eigenvalues of L, hence $\lambda_1 + \lambda_2 + \lambda_3 = a_{11} + a_{22} + a_{33}$ and $\lambda_1 \lambda_2 \lambda_3 = 1$. It follows that the possibilities of the eigenvalues λ_1, λ_2 and λ_3 are : $1, \lambda, \bar{\lambda}$ or $1,1,1$ or $1,1,-1$. Thus the eigen-space of 1 is either 2 or 4.

3-**Theorem 2**

If L is a time-sense preserving Lorentz 4 x 4 matrix with det L = 1 and if 1 is an eigenvalue of order 2, then its eigenspace cannot be tangent to the null cone.

Proof. The coordinates can be choosen in such away that the eigenspace and the eigenvectors are as follows:

$$\left.\begin{matrix} t = z \\ x = y = o \end{matrix}\right\}, \quad \left.\begin{matrix} (0,0,1,1) \\ (1,0,0,0) \end{matrix}\right\} \quad \text{respectively.}$$

For a proper time-sense preserving Lorentz matrix L (i.e., det L = 1) with the eigenvector (0,0,1,1), we have $a_{13} = a_{23} = o$, $a_{33} = 1$, and with the eigenvector (1,0,0,0) we have $a_{21} = a_{31} = o$, $a_{11} = 1$, On denoting

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by F the 4-dimensional diagonal matrix all of whose diagonal elements are 1, save the last, which is -1 and since L is a Lorentz matrix we have $L^* FL = F$. This leads to the following relations : $1 + a_{12}^2 = 1$, $a_{22}^2 = 1$, $a_{22}\, a_{32} = o$, which implies that $a_{12} = o$, $a_{22} = \pm 1$, $a_{32} = o$.

Finally, since $\det L = 1$, then $a_{22} = 1$ and so L will be a unit matrix and hence, the eigenspace cannot be tangent to the null cone.

From the above two theorem< it follows that the folding planes passing through the origin is transverse to the null cone, each folding plane P such that o, x P has the property that $f(y) = y$ for all y P. In other words the 2-folding planes of any isometric folding of Minkowski space into itself keeping the origin fixed can not be tangent to the null cone.

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فراغ منكوفسكى ومستويات طيه

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مصر

فى هذا البحث أثبتنا أن مستويات الطى لأى طى متقايس لفراغ منكوفسكى الى نفسه بحيث تحفظ نقطة الاصل تقطع المخروط الضوئى فى الفراغ ولا تمسه.