الفرقة: الرابعة (ش.الرياضيات) دوريناير ٢٠١٥ المادة: التحليل الدالي رمراطر ١٥١١ الزمن: ساعتان

جامعة طنطا كلية العلوم

# أجب عن الأسئلة التالية:

- 1) a. Define M.S X and give 3 examples. b. Define H.S X and give 2 examples.
- 2) a.  $x_n \to x$   $\iff$   $\cdots$   $\iff$   $\cdots$

c. P.t  $d(x,y) = \sqrt{|x-y|}$  is dist. F in R.

- 3) a. Give 3 examples of N.S X. b. P.t x  $|y \iff ||x - 3y||^2 = ||x||^2 + 9 ||y||^2$ .
- 4) a.  $||x 3y||^2 + ||x + 3y||^2 = \dots + \dots$ b. (x-3y), x+3y) = ... - ... . c. Solve I.E  $y^2(x) = 25 - 6 \int_2^x y(t) dt$ .
- 5) a. P.t F(x) = (x,y) is B.F L.F C.F in H.S X. b. P.t  $Ay(x) = C + \frac{3}{5} \int_0^x y(t) dt$  is contra. Op in C [0,1].

مع أطيب التمنيات بالنجاح .. د .سعيد أحمد أبوالعلا واللجنة





# TANTA UNIVERSITY FACULTY OF SCIENCE

### DEPARTMENT OF MATHEMATICS FINAL EXAM FOR FOURTH LEVEL (MATH STUDENTS)

COURSE TITLE: NUMERICAL ANALYSIS (2)

COURSE CODE: MA4109

DATE: 31/12/2014

TERM: FIRST

TOTAL ASSESSMENT MARKS: 150

TIME ALLOWED: 2 HOURS

#### Answer the following questions:

(I) (a) Find the general solution for the following difference equation:

$$f(x+4) - \frac{5}{4} f(x+3) + \frac{5}{2} f(x+1) - f(x) = 1 ;$$
  

$$f(0) = 0, f(1) = 11, f(2) = -8, f(3) = 6 .$$
 (20 marks)

(b) Discuss the stability for the following numerical procedure with respect to its initial values:

$$y_n - y_{n-3} = \frac{3h}{8} [f(x_{n-3}, y_{n-3}) + 3f(x_{n-2}, y_{n-2}) + 3f(x_{n-1}, y_{n-1})], n \ge 3.$$
 (20 marks)

(II) (a) Apply the method of nets to compute the numerical solutions for the following mixed

problem: 
$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = t + x - 1,$$

$$u(x, 0) = \ln(3 - x), \quad \frac{\partial u(x, 0)}{\partial t} = 0 \quad (0 \le x \le 1);$$

$$u(0, t) = \ln 3, \quad u(1, t) = \ln 2, \quad h = 0.05.$$
(20 marks)

(b) By the method of finite differences, find the numerical solutions for the following linear boundary value problem:

$$y'' + (1 + x^2)y' - xy = 0$$
,  $x \in [0, 1.5]$ ;  $y(1) = 0$ ,  $y(1.5) = 1.62$ ,  $h = 0.01$ . (15 marks)

(III) (a) Define: An interior nodal point - The collocation points - The iterated and the degenerate kernels - The Dirichlet problem. (15 marks)

(b) Find an approximate solution for the following Fredholm's integral equation:

$$x(t) = t^2 - 2 + \int_0^1 (s+1) e^{-ts^2} x(s) ds , t \in [0,1].$$
 (20 marks)

(IV) (a) Making use the collocation method, derive an approximate analytical solution for the following boundary value problem:

$$x^2y'' - xy' + 2y = x^2$$
,  $x \in [-1, 0]$ ;  $y(-1) = y(0) = 0$ . (20 marks)

(b) Define ill and well conditioned problems and whence discuss the stability of the following initial value problem with respect to its initial condition:

$$y'(x) = -[y(x)]^2$$
,  $0 \le x \le b$ ;  $y(0) = 1$ . (20 marks)

EXAMINERS PROF. DR. A. R. M. EL-NAMOURY DR. M. A. SERWAH

جامعة طنطا امتحان الفصل الدراسي الأول المستوى ألثالث-علوم الحاسب كلية العلوم ٢٠١٥-٢٠١ المادة: توبولوجي وجبرحديث (MA4121)

الزمن:ساعتان

قسم الرياضيات

Modern Algebra.

Answer the following:

- Show that Z[x], the set of all polynomials in an indeterminate x with integral coefficients, is a commutative ring with no zero divisors. (10 marks).
- 2- Let  $\varphi: (C; +, .) \to (M_2(R)); +, .)$

 $\mathbf{a}+\mathbf{i}\mathbf{b} 
ightarrow \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$  be a map from the field of complex numbers into the ring of all square matrices of degree 2 with real entries. Show that  $\varphi$  is a ring homomorphism. Find the kernel of  $\varphi$ . Is  $\varphi$  onto?.(10 marks).

- 3- Prove that if  $\varphi$  is a ring homomorphism, then  $\ker \varphi$  is an ideal. (10 marks).
- 4- Determine all ideals that contain the integer 6, and those that contain the integer 8, then deduce the ideal generated by the two integers {6,8}. (10 marks).
  - 5- Find out the zeros of the polynomial  $x^4 + 4x^3 + 6x + 2x + 3 \in \mathbb{Z}_8$  [x], then factorize the polynomial as a product of two polynomials of less degrees. (10 marks).

WiThBeStWisHes



~ We !

## TANTA UNIVERSITY FACULTY OF SCIENCE DEPARTMENT OF MATH.

### APPLIED EXAMINATION FOR FOURTH YEAR STUDENTS OF QUANTUM MECHANICS



989

COURSE TITLE:

QUANTUM MECHANICS

COURSE CODE: MA4115

DATE: JANUARY, 2015

TERM: 1

TOTAL ASSESSMENT MARKS: 100

TIME ALLOWED: 2H.

(20 mark)

- 1. (a) In a three dimension Cartesian coordinate system, is there is any possibility to obtain the eigenfunctions and the corresponding eigenvalues for a particle of mass m vibrates about the center under the influence of attracted force, which is proportional to the distance from an equilibrium position?. (15 mark)
  - (b) Prove that the eigenvalues of the operators  $L_z$  is and  $m\hbar$ . (10 mark)
- 2. (a) Drive the matrix formulae of the operators  $\hat{P}_x$ ,  $\hat{P}_x^2$ ,  $\hat{X}$  and  $\hat{X}^2$  for a particle having a simple harmonic motion (pure state), apply the matrix element to find  $\langle x \rangle$  and  $\langle P_x \rangle$ . (20 mark)
  - (b) Complete the following:  $|\infty\rangle + |\beta\rangle$  if and only if ... (5 mark)
- 3. (a) Show that the uncertainty principle for the wave function  $\Psi(x,t) = \frac{1}{\sqrt{2}} \left[ \Psi_0(x,t) + \Psi_1(x,t) \right]$  is verified, where the matrix elements of the
  - (b) Operation on a ket vector giving:
    - (1) The same ket (2) Different ket (3) Both (1) and (2) (5 mark)
- 4. (a) Discuss the approximate method for solving the Schrodinger wave equation briefly. (15 mark)
  - (b) prove that the angular momentum  $\Omega$  is given by:

$$\Omega = \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \sin \theta \frac{\partial}{\partial \theta} + \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \phi^2}.$$
 (10 mark)

Best Wishes,

Hamiltonian of the linear harmonic oscillator are known.

Prof. Dr. Ahmed E. Aboanber





1949	TANTA UNIVERSITY FACULTY OF SCIENCE - MATHEMAICS DEP.				
	FINAL EXAM FOR FOURTH LEVEL STUDENTS (MATHEMATICAL STATISTICS)				
	COURSE TITLE: TOPOLOGY		COURSE CODE: MA4121	TIME ALLOWED: 2 HOURS	
DATE: 6/1/2015 TERN		TERM: FIRST	TOTAL ASSESSMENT MARKS: 150		

#### Answer the following questions:

1. (a) Let  $(X, \tau)$  be a topological space and  $A, B \subset X$ . Show that

$$(A \cup B)' = A' \cup B'$$
. (20 marks)

- (b) Show that the intersection of an infinite number of open sets in  $\mathbb{R}$  is not necessarily an open set. (15 marks)
- 2. (a) Consider  $X = \{a, b, c, d, e\}$  with topology  $\tau = \{X, \emptyset, \{a\}, \{c, d\}, \{a, c, d\}, \{b, c, d, e\}\}$  and  $A = \{c, d, e\}$ . Find  $A', A^o, \overline{A}, A^b, ex(A)$ , the neighborhood system of e and the relative topology  $\tau_A$  on A. (25 marks)
  - (b) Let  $X = \{a, b, c, d, e\}$  and  $S = \{\{a, b, c\}, \{c, d\}, \{d, e\}\}$ . Find the topology on X generated by S. (10 marks)
- 3. (a) Let  $(Y, \tau_Y)$  be a subspace of  $(X, \tau)$ . If  $\beta = \{B_i : i \in I\}$  is a base for  $\tau$ , prove that  $\eta = \{Y \cap B_i : i \in I\}$  is a base for  $\tau_Y$ . (20 marks)
  - (b) Let  $(X, \tau)$  be a topological space and  $A \subset X$ . Show that

$$\overline{(H \cap \overline{A})} = \overline{H \cap A}$$
 where  $H \in \tau$ . (20 marks)

- 4. (a) Let f be a mapping from a space  $(X, \tau)$  into a space  $(Y, \sigma)$ , show that :
  - i) f is continuous if and only if  $\forall x \in X$  and  $\forall W \subset Y$  is a neighborhood of  $f(x) \implies \exists V \subset X$  is a neighborhood of x such that  $f(V) \subset W$ . (15 marks)
  - ii) f is open if and only if  $f(A^\circ) \subset (f(A))^\circ \ \forall \ A \subset X$ . (15 marks)
  - b) Let X, Y, and Z be topological spaces. If  $f: X \to Y$  and  $g: Y \to Z$  are continues. Prove that  $g \circ f: X \to Z$  is continues. (10 marks)

EXAMINERS	DR. M. A. SERWAH	DR. O. A. EMBABY	
-----------	------------------	------------------	--



Tanta University
Faculty of Science
Department of Mathematics

Final term exam for the first semester 2014-2015

Course title:

Operations Research (1)

Course code: MA4105

1969

Date: 15 /1 /2015

Total Marks: 150

Time allowed: Hours

#### Answer all the following questions:

#### First question: (40 Marks)

- (a) Prove that The positive semidefinite quadratic form f(X) = X CX is a convex function for all X in  $R^n$ .
- (b) Using the Kuhn-Tucker conditions to obtain the optimal solution of the following NLPP  $\min f = x_1 + \frac{4}{x_1} + \frac{x_2}{40x_1} + \frac{5x_1}{8x_2}$  subject to  $x_1 + x_2 \le 11, x_1, x_2 \ne 0$ .

#### Second question: (40 Marks)

(a) Define separable function-separable programming and then formulate the approximate LPP for the following separable programming

max 
$$Z = 3x_1^2 + 2x_2^2$$
 subject to  $x_1^2 + x_2^2 \le 9$ ,  $x_1 + x_2 \le 3$ ,  $x_1, x_2 \ge 0$ .

- (b) Show that  $f(X) = x_1^2 + 4x_2^2 + 3x_3^2 4x_1x_2$  is positive semidefinite.
- (c) Find the local extrema  $f(x_1, x_2, x_3) = x_1^2 + (x_1 + x_2)^2 + (x_1 x_3)^2$ .

#### Third question: (30 Marks)

- (a) Minimize  $f(X) = x_1^2 + x_2^2 + 2x_1 + 4x_2 + 5$  using the steepest descent method starting at the point  $x_1 = 0$  and  $x_2 = 0$ .
- (b) By direct substitution method solve the following NLPP

min 
$$f(X) = f(x_1, x_2) = 4x_1^2 + 5x_2^2$$
 subject to  $2x_1 + 3x_2 = 6$ .

#### Fourth question: (40 Marks)

(a) Show that the following function is convex, concave or neither:

$$f\left(x,v\right) = xe^{x+y}$$

(b) Obtain the necessary conditions for the optimum solution of the following NLPP

by Lagrange multiplier method: min 
$$f(x_1, x_2) = 3e^{2x_1+1} + 2e^{2x_2+5}$$

$$g(x_1, x_2) = x_1 + x_2 - 7 = 0$$

(Best wishes)

- 6



# TANTA UNIVERSITY FACULTY OF SCIENCE DEPARTMENT OF MATHEMATICS

EXAMINATION FOR (FOURTH YEAR) STUDENTS OF MATHEMATICAL

COURSE TITLE: DIFFERENTIAL GEOMETRY

COURSE CODE: MA4107

DATE:19\1\2015

TERM: 1

TOTAL ASSESSMENT MARKS: 150

TIME ALLOWED: 3 HOURS

#### **Answer the Following Questions:**

#### **QUESTION 1:**

a) Find the involutes of the unit circle.

(20)

b) For a space curve  $\alpha(t)$  find k(t) in terms of the derivatives of  $\alpha(t)$ . (20)

#### **QUESTION 2:**

State and prove the isoperimetric inequality. Find the upper bound of the area included by a simple closed curve of length  $l=2\sqrt{\pi}$ . (30)

#### **QUESTION 3:**

a) Prove that the matrix  $(g_{ij})$  is non – singular.

(20)

b) Find the area of the tours  $\chi(t,\theta) = ((a+r\cos t)\cos\theta, (a+r\cos t)\sin\theta, r\sin t)$ 

$$0 < t < 2\pi$$
 ,  $0 < \theta < 2\pi$ . (20)

#### **QUESTION 4:**

a) Show that the sphere  $x^2 + y^2 + z^2 = 1$  is a surface.

(20

- b) Find the equation of the tangent plane to the surface  $\chi(u^1, u^2) = (u^1, u^2, u^1u^2)$
- at the point  $u^1 = a, u^2 = b$ .

(20)

**EXAMINERS** 

PROF. DR. M. BELTAGY

PROF.DR. SANAA ELASSAR

With best wishes