

First Semester Exam. (Jan. 2013)

Please answer the following questions:

1 (a) Derive the surface conditions for a stresses acting on area inclined to the co-ordinate planes

(b) For a stress tensor  $T = \begin{pmatrix} 7 & 0 & -3 \\ 0 & 4 & 0 \\ -3 & 0 & 6 \end{pmatrix}$   
Calculate:  $H, \bar{H}, \Delta, N_i$  and  $l_i, m_i, n_i$

2 Derive  $T_{uu}, T_{uv}$  and  $T_{uw}$  in general form and then in plane  $yx$  at  $x = \frac{\pi}{2}$ . Calculate  $T_{uu}, T_{uv}$  and  $T_{uw}$ . [Plot your answer]

3 (a) Derive  $e_{xh}, e_{yh}, e_{zh}$  in terms of  $u, v$  and  $w$

(b) when  $u = xyz^2, v = y^2z$ , and  $w = x^2y^2$  Calculate  $e_{xh}, e_{yh}$ , and  $e_{zh}$  at point  $(1, 2, 1)$ ,  $l = m = n$

4 (a) Derive the differential eq<sup>s</sup> of equilibrium in plane  $(r, \theta)$

(b) On the basis of Fourier's method, prove that the longitudinal vibrations of a Bar satisfy  $\frac{\partial^2 u}{\partial t^2} = \frac{E}{\rho} \frac{\partial^2 u}{\partial x^2}$

5 (a) On the basis of elasticity principals prove that  $T_{xx} = \lambda \theta + 2\mu e_{xx}$

$$T_{yy} = \lambda \theta + 2\mu e_{yy}$$

$$T_{zz} = \lambda \theta + 2\mu e_{zz}$$

(b) Calculate Lamé's coefficients  $\lambda, \mu, E, K$  when  $\nu = 0.3, u = -2x + 1, v = 3y + 1$

and  $T = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 4 \\ 3 & 4 & 0 \end{pmatrix}$

with best wishes.

**Answer the following questions**

1- (a) Prove that if  $(X, Y)$  has a bivariate normal distribution, then  $X$  and  $Y$  are independent if and only if  $X$  and  $Y$  are uncorrelated.

(b) Given two random variables (r.v.'s)  $X$  and  $Y$  with joint probability mass function

	x	0	1	2
y				
0		0.1	0.1	0
1		$K$	0.2	0.1
2		0	0.1	0.1
3		0.1	0	0.2

Find:  $K$ , the correlation coefficient  $\rho(X, Y)$  and  $E(X | Y = 2)$ .

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2-(a) Find the characteristic function of the random variable  $Y = (X - np) / \sqrt{npq}$ , where  $X$  follows the binomial distribution.

(b) Suppose that the joint probability density function (p.d.f.) of  $X$  and  $Y$  is given by

$$f(x, y) = 4y(x - y)e^{-x-y}, \quad 0 \leq x < \infty, \quad 0 \leq y \leq x.$$

Compute  $E(X | Y = y)$ .

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3- (a) If the probability mass function (p. m. f.) of the random variable  $X$  is

$$f_x(x) = \frac{3}{4} \left(\frac{1}{4}\right)^{x-1}, \quad x = 1, 2, 3, \dots, \text{ then find the p. m. f. of } Y = X^2.$$

(b) If the joint probability mass function of the discrete r. v.'s  $X$  and

$$Y \text{ is } f(x, y) = \frac{\lambda^y}{(y-x)! x!} e^{-2\lambda}, \quad x = 0, 1, 2, \dots, y \text{ and } y = 0, 1, 2, \dots,$$

then calculate  $f_y(y)$  and the conditional variance of  $X$  given that  $Y = 3$ .

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4- (a) If  $X_1, X_2, \dots, X_n$  are independent and identically distributed r.v.'s having an exponential distribution with parameter  $\mu$ , then find the distribution of

$$Z = X_1 + X_2 + \dots + X_n.$$

(b) Let  $X_1$  and  $X_2$  be a random sample of size 2 from the distribution having p. d.f.  $f(x) = e^{-x}, \quad x > 0.$

Find the p. d. f. of  $Y = X_1 + X_2$  and the p. d. f. of  $Z = \frac{X_1}{X_1 + X_2}.$



TANTA UNIVERSITY  
FACULTY OF SCIENCE  
DEPARTMENT OF MATHEMATICS

EXAMINATION FOR PROSPECTIVE STUDENTS (4<sup>TH</sup> YEAR) STUDENTS OF MATHEMATICS

COURSE TITLE: GENERAL RELATIVITY & ELECTRODYNAMICS COURSE CODE: 14016

DATE:16

JAN, 2013

TERM: FIRST

TOTAL ASSESSMENT MARKS: 63

TIME ALLOWED: 3 HOURS

**Answer 5 questions only:**

1- i) Consider the boundary between free space and a perfect dielectric having  $\epsilon_r = 9, \mu_r = 1$  and  $\sigma = 0$ . If a uniform plane wave  $\underline{E}_I = \cos(\omega t - \frac{4\pi}{3}z) \hat{i}$  and frequency of 200MHz is incident from free space normal to the dielectric. Find the time domain forms of the reflected and transmitted fields.

ii) Consider an electric field  $\underline{E} = E_0 r e^{-at} \hat{k}$ , where  $E_0$  is constant. Find the magnetic field produced by this varying field.

2- Derive the reflection and transmission coefficient through linear media.

3- Discuss the TM waves guided by a rectangular guide.

4- Discuss Einstein field equations, then by using the Bianchi identity:

$$R_{ijk//l}^n + R_{ikl//j}^n + R_{ilj//k}^n = 0, \quad \text{Prove that} \quad G_{j//i}^i = 0$$

5- i) Using  $(A_i B_j)_{//k} = A_i (B_{j//k}) + (A_{i//k}) B_j$  determine  $A_{ij//k}, (A_{i//j})_{//k}$

ii) Prove that the derivative of a scalar function is a covariant vector (tensor).

iii) Show that  $g^{\alpha\beta} g_{\alpha\beta} = n$  for n-dimensional space.

6- Prove that i)  $\Gamma_{jk}^i = \frac{\partial}{\partial x^k} \ln \sqrt{g}$

ii)  $[p q, r] = [q p, r]$

iii)  $[p q, r] = g_{rs} \Gamma_{pq}^s$

EXAMINERS	PROF. DR/MOHAMED O. SHAKER	DR/ MOHAMED M. SHAHIN
	DR/ ABDALLAH A. NAHLA	DR/

*With my best wishes*

