وحـدة ضمـان الجـودة
كلية العلوم - جامعة طنطا
QUALITY ASSURANCE UNIT
FACULTY OF SCIENCE - TU

ACCREDITED 2016
1969

المستوى الثالث

احصاء

1969	Tanta University Faculty of Science Department of Mathematics		
	Final term exam for the Second semester 2016-2017		
	Course title:	Optimal Control +Functional Analysis	**Course code: MA3210**
	Date:4 /6/2017	Total Marks: 150	**Time allowed: 2** Hours

Answer all the following questions:

First question: (45 Marks)

(a) Show that the set $S=\{(x_1,x_2)\in R^2 : |x_1|+|x_2|<1\}$ is convex set.

(b) Find a necessary condition for a function to be an extremal for the functional $J(x)=\int_{t_0}^{t_f} F(x(t),x'(t),t)\,dt$, where $t_0, x(t_0), t_f\ x(t_f)$ are specified?.

(c) Show that:

(i) Euler's equation can be written in the form: $\frac{d}{dt}\left(F-x'\frac{\partial F}{\partial x'}\right)-\frac{\partial F}{\partial t}=0$.

(ii) If F is not an explicit of t, then $F-x'\frac{\partial F}{\partial x'}=c$.

Second question: (30 Marks)

(a) On what curves can the functional:

$$J[x(t)]=\int_1^2\left(x'^2-2xt\right)dt,\quad x(1)=0,\quad x(2)=-1$$

attain an extremum.

(b) Optimize $J(x)=\int_{t_0}^{t_f}\left(2x'^2-42xt+11t\right)dt$ s.t. $x(t_0)=x_0$ and $x(t_f)=x_f$

(c) Find the extremals of the problem corresponding to the functional

$J=\int_0^{\frac{\pi}{2}}\left(2x_1x_2-2x_1^2+x_1'^2-x_2'^2\right)dt$ s.t. $x_1(0)=0, x_1\left(\frac{\pi}{2}\right)=1, x_2(0)=0, x_2\left(\frac{\pi}{2}\right)=1$.

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Third Equestion: (40 Marks)

(a) Define the following:

(1) Bounded set in metric space (X,d).

(2) Convergent sequence in metric space (X,d).

(3) Cauchy sequence in metric space (X,d).

(4) Complete metric space (X,d). (5) Incomplete metric space (X,d).

(6) Contraction mapping (7) Continuous mapping. (8) Linear space.

(9) Normed space $(X,\|\cdot\|)$ (10) Fixed point

(b) State and prove Cauchy's inequality. Hence or otherwise prove that the space (R^n,d) is complete metric space while the metric space $((0,1],d)$ is incomplete metric space.

(c) Prove that the space l_p with $\|x\|=\left(\sum_{j=1}^{\infty}|x_j|^p\right)^{\frac{1}{p}}$ is Banach space.

Fourth Equestion: (38 Marks)

(a) If $f(x)$ and $\varphi(x)\in L_2(a,b)$, moreover $\int_a^b\int_a^b k^2(x,y)\,dx\,dy<\infty$. Prove that the Fredholm integral equation $f(x)=\varphi(x)+\lambda\int_a^b k(x,y;f(y))\,dy$ has a unique point if $|\lambda|<\left(\int_a^b\int_a^b k^2(x,y)\,dx\,dy\right)^{-\frac{1}{2}}$.

(b) Prove that the functions " $d(x,y)$, contraction mapping and $\|x\|$ " are continuous functions.

(c) If $S=\{x:x=(\xi_1,\xi_2,\cdots,\xi_n,\cdots),\xi_i\in R\ \forall j=1,2....\}$ and $d(x,y)=\sum_{j=1}^{\infty}\frac{1}{2^j}\frac{|\xi_j-\eta_j|}{1+|\xi_j-\eta_j|}$.

Prove that:

(1) (S,d) is metric space.

(2) $(S,\|\cdot\|)$ is not normed space, where $\|x\|=d(x,0)$.

(d) In R^n, we define $d_1(x,y)=\max\{|x_j-y_j|\}$, $d_2(x,y)=\sum_{j=1}^{n}|x_j-y_j|$ and $d_3(x,y)=\sqrt{\sum_{j=1}^{n}(x_j-y_j)^2}$. Prove that d_1, d_2, d_3 are equivalent.

Examiners:	1- Prof. Dr. S. Abdel Aziz-	2- Dr. N. El-Kholy

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First question: (45 Marks)

(a) Show that the set $S = \{(x_1, x_2) \in \mathrm{R}^2 : |x_1| + |x_2| < 1\}$ is convex set.

(b) Find a necessary condition for a function to be an extremal for the functional $J(x) = \int_{t_0}^{t_f} F(x(t), x'(t), t)\, dt$, where $t_0, x(t_0), t_f\ x(t_f)$ are specified?.

(c) Show that:

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