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DISTRIBUTION OF VELOCITIES IN AN OSCILLATING GAS BUBBLE

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ABSTRACT

The distribution of velocities inside a spherical gas bubble suffering radial oscillations is studied. The profiles of velocities and gas temperatures in the bubble are calculated and the curve is presented which characterizes the bubble radius variations with time in the case of radial oscillations of the bubble corresponding to its behaviour directly behind the front of a strong shock wave. It is shown that the velocity profile of gas particles in a pulsating bubble may differ noticeably from the linear dependence corresponding to uniform distribution of density in the bubble.

INTRODUCTION

The difficulties when finding velocity profile inside a pulsating bubble, authors of some works treated the dynamics of bubbles in liquids having presenting the distribution of velocities in such bubble as a linear function [3].

$$V\ (r,t) = \dot{a}\ \left(\frac{r}{a}\right) \qquad (1)$$

or of the quadratic dependence [2]

$$V(r,t) = \dot{a}\left(\frac{r}{a}\right)^2 \qquad (2)$$

where a is the bubble radius; r is the radial coordinate measured from the bubble center; V is velocity.

The exact dynamic expression of gas velocity profile in a bubble may be obtained in terms of the admission of uniform pressure in the bubble which is valid if the bubble surface velocity is much below sonic velocity in gas [5]. The set of equations describing the heat exchange and the dynamics of a spherical gas bubble with liquid can be written as

$$\rho_g C_p \frac{dT}{dt} = r^{-2} \frac{\partial}{\partial r}\left(\lambda r^2 \frac{\partial T}{\partial r}\right) + \frac{dP_g}{dt} \qquad (3)$$

$$\int_0^r \frac{d\rho_g}{dt} + \int_0^r \rho_g r^{-2} \frac{\partial}{\partial r}(r^2 V) = 0 \qquad (4)$$

$$P_g = \rho_g B T \qquad (5)$$

$$a\ddot{a} + \frac{3}{2}\dot{a}^2 = \left(P_g - P_\infty - 2\frac{\sigma}{a}\right)\rho_l^{-1} \qquad (6)$$

Here, ρ, P,T are respectively density, pressure, temperature; C_p is the heat capacity of gas at a constant pressure; λ

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is thermal conductivity of gas; B is gas constant; σ is surface tension of liquid. The subscripts g, l, ∞ relate to the parameters of gas, liquid, and far from the bubble, respectively.

The boundary condition, which have to be prescribed for the equation of gas heat influx (3), may be presented as

$$r = 0 : \frac{\partial T}{\partial r} = 0 \qquad V = 0 \tag{7}$$

$$r = a : \quad T = T_o = \text{Const} \ , \quad V = \dot{a} \tag{8}$$

If the gas continuity equation (4) is integrated from 0 to a using the equations of heat influx (3) and of state (5) and employing the admission of uniform pressure in the bubble, we shall obtain the integral of the heat influx equation

$$\frac{dP_g}{dt} = \frac{3(\gamma - 1)}{a} \left(\lambda \frac{\partial T}{\partial r} \right)_a - \frac{3 \gamma P_g}{a} \dot{a} \tag{9}$$

where γ is the adiabatic index of gas.

The integration of the continuity equation from 0 to r makes it possible to find the profile of particle velocities in bubble (3) which takes the following simple form in the absence of phase transitions :

$$V = \frac{r}{a}\dot{a} + \frac{\gamma - 1}{P_g}\left[\left(\lambda\frac{\partial T}{\partial r}\right)_r - \frac{r}{a}\left(\lambda\frac{\partial T}{\partial r}\right)_a \right] \quad (10)$$

The set (3) - (10) was solved numerically by the finitedifference method for dividing the system into spherical layers with introduction of the variable $\eta = \frac{r}{a}$ which freezes the mobile boundary of the bubble.

Consideration was given to the problem of radial pulsations of a bubble in a liquid due to an instantaneous (at t=0) rise of pressure in the liquid far from the bubble from P_0 to P_∞, which corresponds, in particular, to the behaviour of gas bubbles in the beginning of the bubble screen as a shock wave penetrates the latter [5].

Fig.I presents the curve illustrating the temporal behaviour of bubble radius for a_o = 0.01 mm, P_o = 0.1 MPa, P_∞ = 0.2 MPa , $r = tD_g / a_o^2$; $D_g = \lambda / \rho_g C_p$

It is seen that the radial oscillations of gas bubble attenuate markedly due to heat dissipation. The comparison with [5] where a_o = 0.1 mm has shown that a decrease of the inial size of the bubble enhances the attenuation substantialy because of the resultant increase of the specific interphase surface falling at unit gas mass.

Fig. 2 presents the temperature profiles corresponding to the version displayed in Fig. 1 for the gas inside an oscillating bubble at different moments. Curves 1 and 5 are for two successive moments of the maximum bubble compression which are indicated in Fig. 1 with points A and B. If new time made dimensionless with respect to the bubble oscillation period is introduced in the time interval between the points A and B, then the points A and B (or curves 1 and 5) will correspond to the moments $t' = 0$ and 1. Curves 2-4 are for the moments $t' = 1/4$, 5/12 and 7/12.

The comparison of the curves in Fig. 2 with the respective curves for the larger bubble $a_o = 0.1$ mm presented in [5] has shown that a decrease of the initial size of the bubble gives rise to a strong flattening of the curves characterizing the local temperature distribution in a gas bubble.

Fig. 3 shows the curves corresponding to Fig. 2 and illustrates the distribution of gas particle velocities in a radially-oscillating gas bubble at different moments of time. The notations for the curves is the same as in Fig. 2 with the dimensionless velocity being $W = V/\eta \dot{a}$. The linear distribution of velocity [3] corresponds to W = 1 (the dashed curve), and the parabolic profile (2) recommended in [1] to the dash-dotted straight line. From Fig. 3 it is seen that the gas particle velocity profile in a pulsating bubble differs noticeably from both linear and quadratic dependences.

The difference of the velocity profile from linear dependence in the particular version discussed above reaches 40%.

In case of small bubbles (R < 1 mcm), the temperature dependence is in practice uniform because their oscillations follows the isothermic mode. In this case the velocity distribution in the bubble is nearly linear.

CONCLUSIONS

The velocity distribution inside a spherical gas bubble suffering radial oscillations has been studied quantitatively. It has been shown that the gas particle velocity distribution in an oscillating bubble may prove to be very different from the linear dependence corresponding to the model of homogeneous bubble. In case of very small bubbles ($R \leq 1$ mcm) the velocity distribution in an oscillating bubble is in practice linear. The quadratic dependence proposed in [1] for the gas particle velocity profile in a radially-oscillating bubble is not more accurate compared with the linear approximation.

REFERENCES

1- Cho S.M., Seban R.A. On some aspects of stream bubble collapse J. Heat Transfer. Trans. ASME, Ser. C., 1969, v. 91, No. 4,

2- Fanelli M., Prosperetti A., Reali M. Radial oscillations of gas-vapour bubbles in liquids. Part 1. Mathematical formulation. - Acustica, 1981, v. 47, 253 - 265.

3- Flinn. G.G. Dynamics of cavitational bubbles set in motion at a large amplitude by acoustic field of pressure. Mechanics; Periodic Collection of Translated Papers, 1976, No. 6.

4- Gubaidullin A.A. et al. Waves in liquids with bubbles. In : Results of Science and Technology, ser. Mechanics of Liquids and Gases, 1982, v. 17.

5- Nigmatulin. R.I. Principles of the Mechanics of Heterogeneous Media, Nauka, Moscow, 1978.

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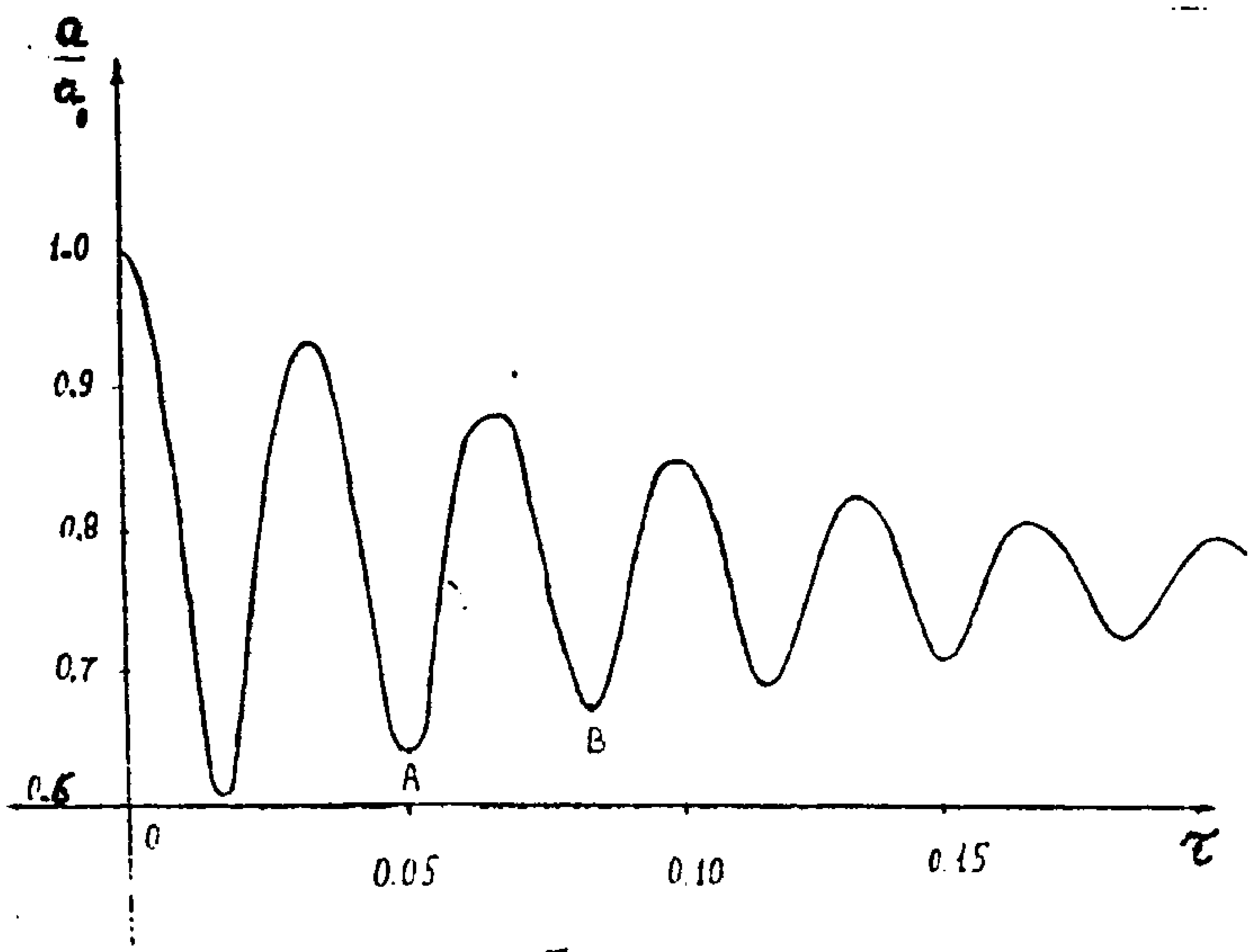

Fig.1

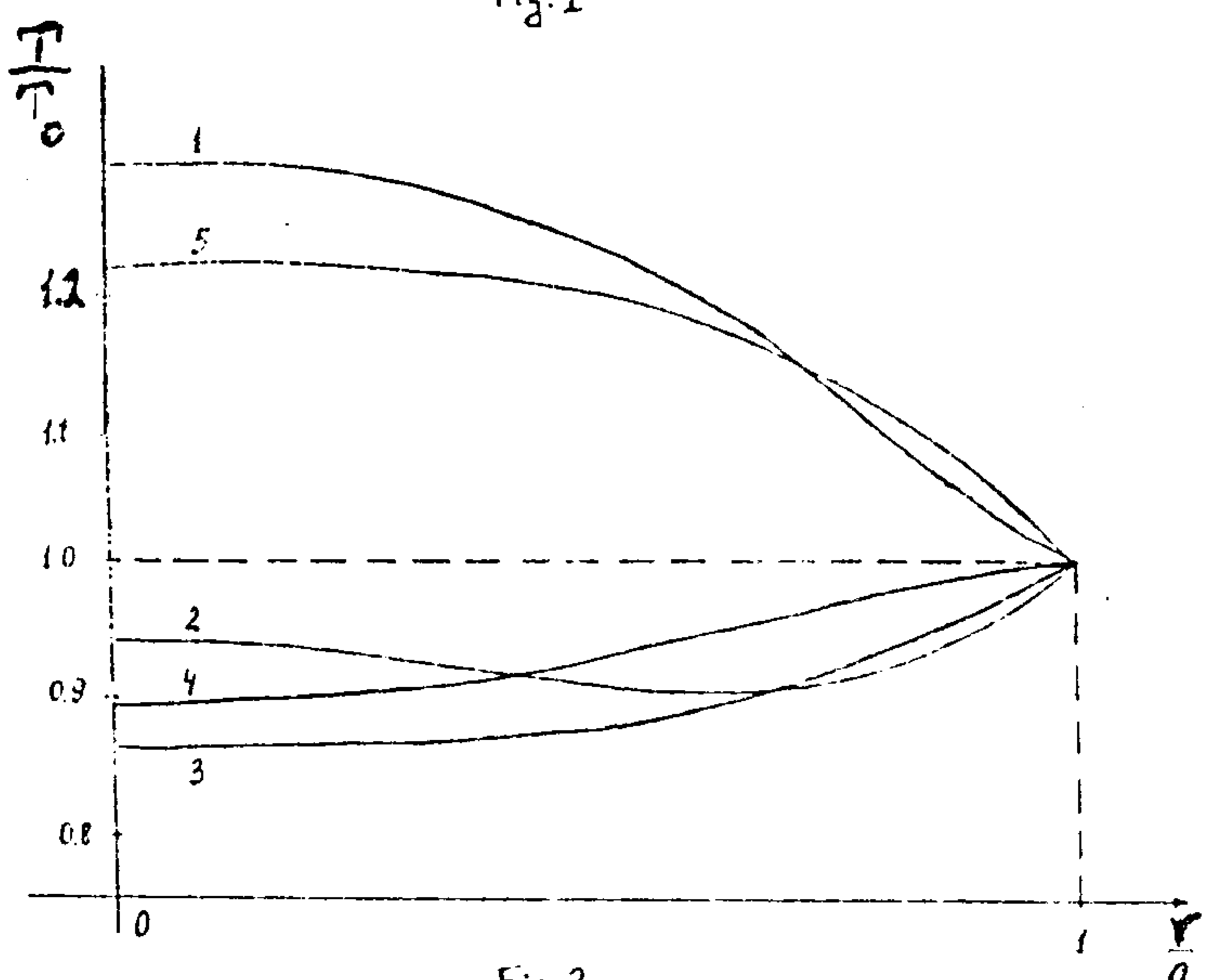

Fig.2

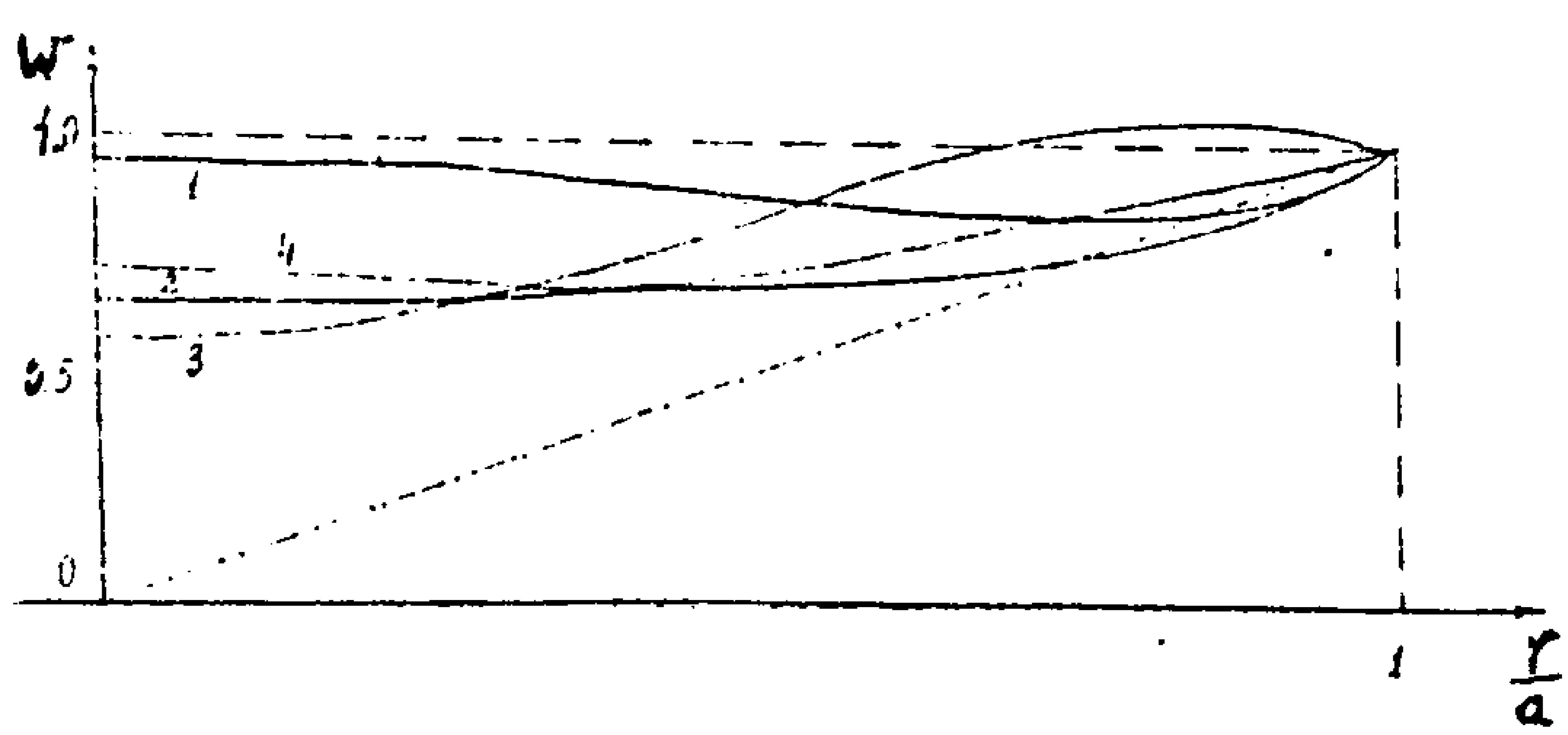

Fig.3

توزيع السرعات لفقاعه غازيه عند التاثير عليها بضغط مفاجىء

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تم دراسه وحساب كل من السرعات ودرجة الحراره داخل الفقاعه الغازيه وتغيرها مع الضغط المفاجىء٠ تم رسم المنحنيات التى تبين تغير نصف القطر مع الزمن بعد التاثير على الفقاعه بذبذبه فجائيه فى اتجاه نصف القطر ٠