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COLLISION μ^+ – MESONS WITH DEUTERIUM MOLECULAR IONS $(H_1^2)_2^+$

By

Sabbah A. S. and Ramadan S. Tantawy

Math. Dept., Faculty of science, Zagazig university

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ABSTRACT

The differential cross section of the chemical reaction

$$\mu^+ + (H_1^2)_2^+ \rightarrow (\mu H_1^2)^+ + (H_1^2)^+$$

has been investigted using the distorted wave approximation. The wave function describing the complete system has been expressed in terms of six wave functions corresponding to the internal, viberational and the relative motion wave functions of both the initial and final states. The calculated results have been compared with the maximum theoretical cross section (π / k^2), where k is the initial wave number for the relative motion of μ^+.

INTRODUCTION

The problems including mesons are interested for scientists, the possibility of simultaneous description of light and heavy mesons in a potential model was discussed [1] and the probability of capture of mesons (π^-) by nuclei was calculated [2].

The scattering of μ^+ mesons with molecules is also very interesting for scientists [4], [3]. With the presence of matted, μ^+ mesons will undergo chemical reactions that may depolarize it and the extent of these depolarizations may be determined by the change in the angular correlation with the positron and tritium nucleus emissions in decay respectively.

The aim of this paper is to study theoretically the rate of chemical reactions of μ^+ mesons with deuterium molecular ions. For this purpose the differential cross section is derived in the framework of the distorted wave differential cross section is derived in the framework of the distorted wave approximation for a collision in which μ^+ mesons are incident on $(H_1^2)_2^+$ and a rearrangement occurs in which μ^+ meson is captured and $(H_1^2)^+$ is emitted leaving $(\mu H_1^2)^+$.

FORMULATION OF THE PROBLEM

The wave equation for the complete system may be written either in terms of the initial or final state [4], [5], i.e.,

$$[H(\bar{r},\bar{\rho})-\frac{\hbar^2}{2M_1}\nabla_\rho^2+v_1-E]\Psi=0 \tag{1}$$

or

$$[H'(\bar{r}',\bar{\rho}_a)-\frac{\hbar^2}{2M_2}\nabla_{\rho'}^2+v_2-E]\Psi=0 \tag{2}$$

where H, H' are the Hameltonians for the internal motion of $(H_1^2)_2^+$ and $(\mu H_1^2)^+$ respectively, E is the energy of the system and v_1 are v_2 the interaction energies dealing with the reduced masses M_1, M_2 of μ^+ with $(H_1^2)_2^+$ and $(H_1^2)^+$ with $(\mu H_1^2)^+$ respectively.

The wave function ψ describing the complete system has been expressed in terms of six wave functions, i.e.,

$$\Psi=\psi(r,R)\chi(R)F(\rho)+\phi(\rho_a,r')\Omega(\rho_a)G(\rho') \tag{3}$$

where $\psi(r,R)$ and $\phi(\rho_a, r')$[6] are the two internal wave functions dealing with the electron motion in the field of the $(H_1^2)_2^+$ and in the $\mu^+, (H_1^2)^+$ field resprectively, the viberational wave functions χ and Ω [3] describe the ground state of $(H_1^2)_2^+$ and $(\mu H_1^2)^+$ molecules. Finally the rewlative motion wave functions F and G deal with μ^+ in the field of $(H_1^2)_2^+$ and deuteron in the field of $(\mu H_1^2)^+$.

Now, we are going to calculate the differential cross section for the transition from initial to final states.

From (1), (2) and (3) we get,

$$\left[-\frac{2M_1}{\hbar^2}(\nabla_\rho^2 + k_1^2) - \bar{v}_1 \right]F =$$

$$= \int \Psi \chi \left[-V_2 + e^2 \left(\frac{1}{\rho_b} + \frac{1}{R} - \frac{1}{r_b}\right) \right] \phi \Omega G_\circ \, d\bar{r} d\bar{R} \quad (4)$$

and

$$\left[\left(\frac{-\hbar^2}{2M_2} \right)(\nabla_{\rho'}^2 + k_2^2) - \bar{V}_2\right] G(\rho') =$$

$$= \int \phi \, \Omega \left[\bar{v}_1 - v_1\right] \psi \, \chi F_\circ d\tau_{(\mu H_1^2)^+} \quad (5)$$

Treating the right hand side of (5) as a known function we have the asymptotic form

$$G(\rho') = \frac{2M_2}{\hbar^2} \frac{e^{ik\rho'}}{\rho'} \left(\frac{-1}{4\pi} \int \phi \Omega G_\circ V_1 \psi \, \chi F_\circ d\, \bar{r}' d\, \bar{\rho}_a d\, \bar{\rho}' \right),$$

where $F_\circ$ and $G_\circ$ are the solutions of the differential equations

$$\left[\nabla_\rho^2 + k_1^2 - \frac{2M_1}{\hbar^2} \bar{v}_1 \right] F_\circ = 0, \quad (6)$$

and

$$\left[\nabla_{\rho'}^2 + k_2^2 - \frac{2M_2}{\hbar^2} \bar{v}_2 \right] G_\circ = 0, \quad (7)$$

Where k_1, k_2 are the wave numbers of the relative motion of μ^+ and $(H_1^2)^+$ respectively, $\bar{v}_1$ and $\bar{v}_2$ are the mean interaction energies and are taken as Morse potential [3], i.e.,

$$V = D(e^{-2\beta(\rho-\rho_\circ)} - 2e^{-2\beta(\rho-\rho_\circ)}),$$

where $D = 1.0154$ e.v, $\beta = 1.2 \ a_\circ^{-1}$ and $\rho_\circ = 2.0 \, a_\circ$

The differential cross section of transition from initial state to final state is defined as

$$Q = \frac{1}{\pi}\frac{M_1 M_2}{m_e^2}\frac{k_2}{k_1}|I|^2, \tag{8}$$

where

$$I = \int \varphi \chi F_o \left\{ \frac{e^2}{\rho_b} + \frac{e^2}{\rho_a} - \frac{e^2}{|\bar{\rho} - \bar{r}|} \right\} \phi \Omega G_o \, d\bar{r}' d\bar{\rho} \, d\bar{\rho}' \tag{9}$$

CALCULATIONS AND RESULTS

The integration in (9) has been simplified by substituting the definitions of the functions Ψ, ϕ, χ and Ω. By using Numerov, Rapits-Allison and the Fourth order Runge-Kutta methods we solve the differential equations in (6) and (7) to obtain F_o and G_o numerically.The energy difference between ground vibrational states is found equals 0.36426 e.v.. The functions F_o and G_o have been normalized and have been illustrated graphically in figures (1), (2), (3), (4), (5) and (6). The differential cross section in equation (8) has been calculated at different energies numerically by using Simpson Rule and the results have been shown in the table:

$k^2 (a_o^{-2})$	$Q_{cal.}$			$Q_{max.}$
	Runge-Kutta	Numerov	Rapt-Allison	
8.365123	0.3742712	0.36753	0.369404	0.3755474
12.81261	0.245186	0.2451163	0.2451376	0.2451883
15.77759	0.199108	0.199102	0.199105	0.1991115

(The unit of the differential cross section is a_o^2)

From the table, it can be seen that the calculated cross sections $Q_{cal.}$ may be compared with the corresponding maximum ones $Q_{max.}$.
In the mean time, it is shown that fourth-order Runge-Kutta method is the most suitable one in our work.

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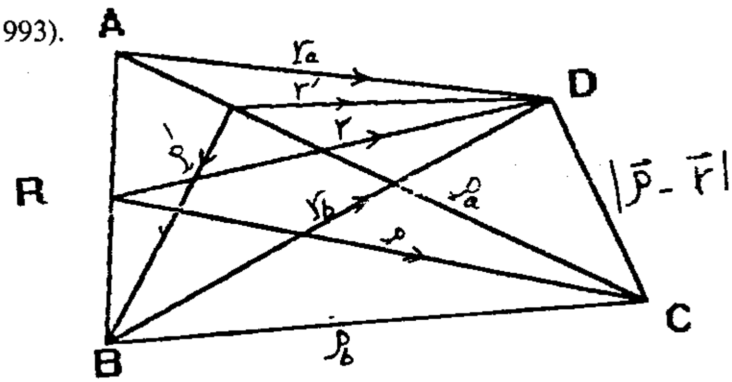

(Definition of the coordinate system used in the calculations where A, B, C and D indicate e^+, e^+, μ^+ and e^- respectively).

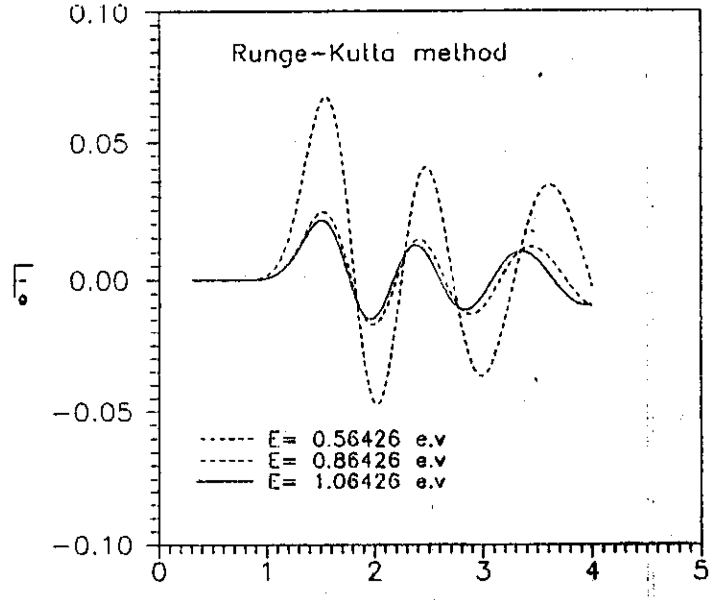
Runge-Kutta method
E= 0.56426 e.v
E= 0.86426 e.v
E= 1.06426 e.v
0.10
0.05
0.00
-0.05
-0.10
0
1
2
3
4
5

Fig.(1)

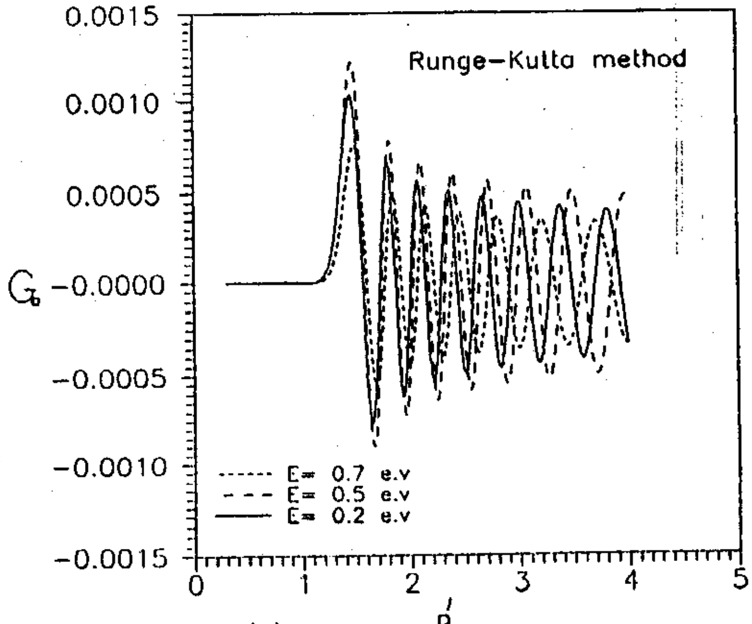
Runge-Kutta method
E= 0.7 e.v
E= 0.5 e.v
E= 0.2 e.v
0.0015
0.0010
0.0005
-0.0000
-0.0005
-0.0010
-0.0015
0
1
2
3
4
5

Fig.(2)

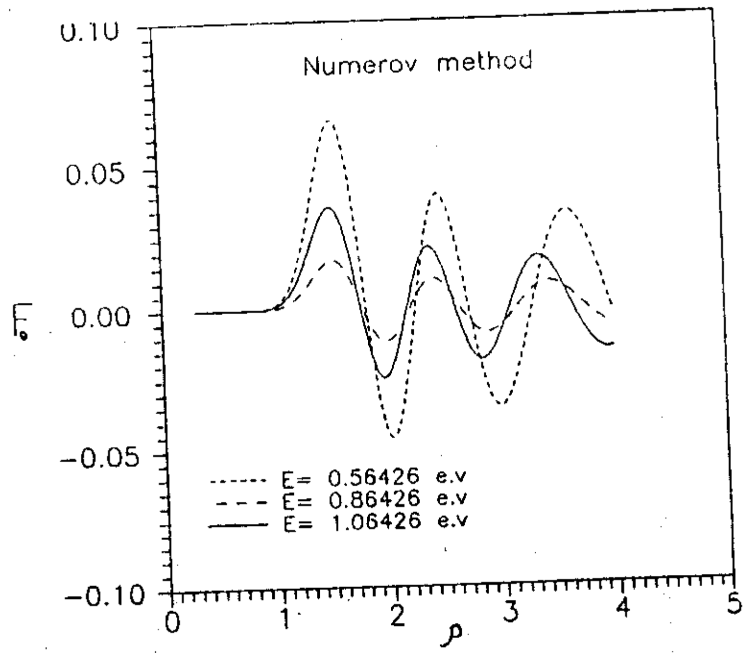
Numerov method
0.10
0.05
0.00
−0.05
−0.10
F₀
E= 0.56426 e.v
E= 0.86426 e.v
E= 1.06426 e.v
0
1
2
3
4
5
ρ

Fig.(3)

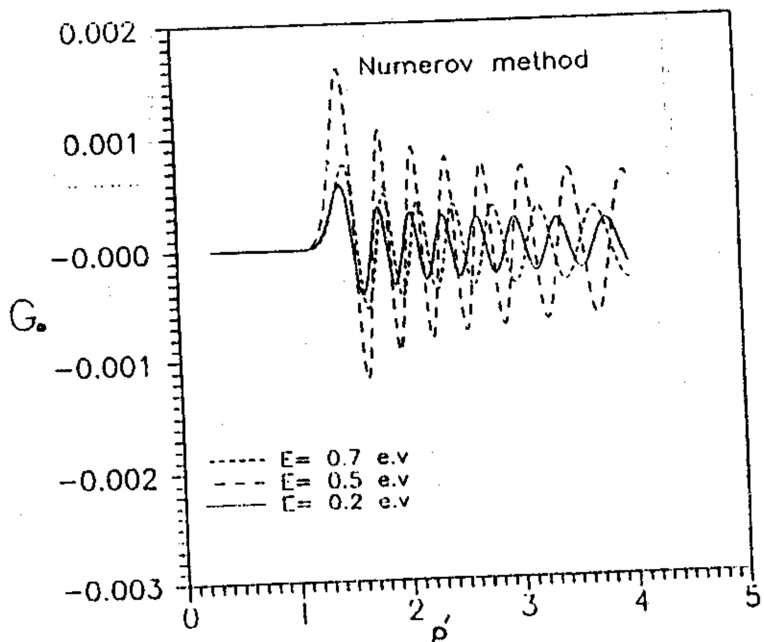
Numerov method
0.002
0.001
−0.000
−0.001
−0.002
−0.003
G₀
E= 0.7 e.v
E= 0.5 e.v
E= 0.2 e.v
0
1
2
3
4
5
ρ'

Fig (4)

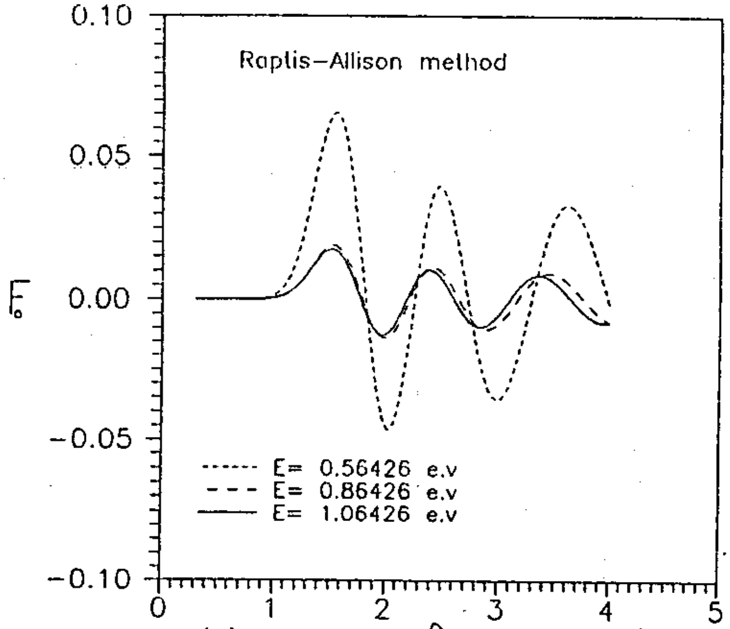
Raplis-Allison method
0.10
0.05
0.00
-0.05
-0.10
F
E= 0.56426 e.v
E= 0.86426 e.v
E= 1.06426 e.v
0
1
2
3
4
5
ρ

Fig.(5)

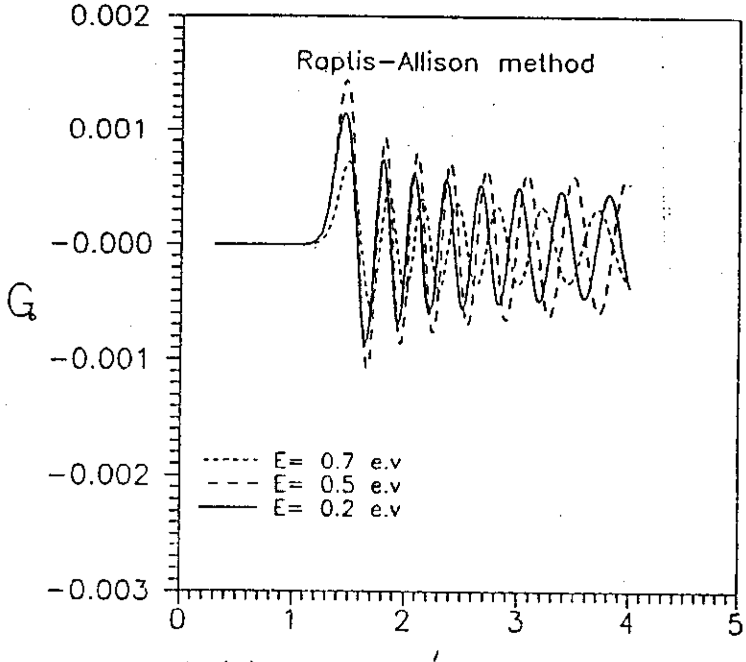
Raplis-Allison method
0.002
0.001
-0.000
-0.001
-0.002
-0.003
G
E= 0.7 e.v
E= 0.5 e.v
E= 0.2 e.v
0
1
2
3
4
5
ρ'

Fig.(6)

تصادم ميوزونات الميو الموجبة مع جزيئات الديتريوم الموثينة $(H_1^2)_2^+$

عبد الله الصباح- رمضان صالح الطنطاوى
قسم الرياضيات---كلية العلوم- جامعة الزقازيق

باستخدام طريقة الموجة المشوهة تم حساب المقطع المستعرض التفاضلى للتفاعل الكيميائى

$$\mu^+ + (H_1^2)_2^+ \rightarrow (\mu H_1^2)^+ + (H_1^2)^+$$

حيث تم التعبير عن الدوال الموجبة التى تصف النظام كاملا بدلالة ستة دوال موجبة مناظرة للحركة الداخلية والحركة الاهتزازية التذبذبية والحركة الموجبة النسبية لكل من الحالة الابتدائية والحالة النهائية. تم مقارنة القيم المحسوبة بالقيم العظمى (π / k^2) المناظرة لها ، حيث k يمثل العدد الموجى الابتدائى لحركة μ^+ النسبية.