

STEADY COUTTE FLOW OF A RAREFIED GAS CONSISTING  
OF CHARGED PARTICLES MOVING IN A CONSTANT  
MAGNETIC FIELD WITH ARBITRARY  
REFLECTION COEFFICIENTS

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ABSTRACT

Plane steady Couette flow at low Mach number is studied in the presence of a constant magnetic field. The particles reflected from the two walls with arbitrary reflection coefficient are considered. The density of our gas is considered as a function of  $x$  ( $x$  is the distance along the walls). An approximate solution to the Boltzmann equation, of modified Liu-Lees type, is found to yield simple analytic expression for flow velocity, shear stress and coefficient of viscosity. These predictions give correct results in both the continuum and rarefied limits and show reasonable agreement with other more exact predictions in the limited regimes for which such predictions have been obtained. We shall neglect electric fields arising from the distribution of charged particles, and induced magnetic fields which will arise from bulk motion of the charged gas.

Introduction

Rarefied gas dynamics problems arise in several disciplines including those the vacuum science and technology, space science, astrophysics, plasma physics and nuclear fusion and fission. Successful formulation and solution of rarefied

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gas dynamics problems is also crucial to advances in aspects of environmental sciences where understanding and control of the formation, motion, reactions and evolution of particles of varying composition and shapes, and their spatial and temporal distribution, under gradients of concentration, temperature, pressure and electromagnetic radiation and other fields is of importance.

The consideration of problems with boundary surfaces is still of modest interest for the case of a plasma. In literature [Shidlovskiy, 1967] [1] the case of Couette flow of fully ionized plasma in an external electric field is presented. The method is based on replacing the Boltzmann equation by moment equations with the simultaneous introduction of two-stream Maxwellian distribution functions.

Simple problem in plane Couette flow, originally suggested by pomraning [2] poses interesting questions in gas dynamic and kinetic theory. A negative-harmonic-oscillator force is imposed on the gas, driving molecules towards either boundary plate.

Earlier work on this problem has been either incorrect [2] (see Ref. 3), or has given predictions for only a limited portion of the circumstances considered here [3-6]. Johnson and Stopford [5] obtained predictions showing noncontinuum effects, using a standard Gross-Jackson-Ziering [7] half-range polynomial expansion to solve the appropriate BGK [8] equation. The method appears to yield predictions, however, only when gas densities and force strengths are such that the gas is near-continuum throughout Cassell and William [6] on the other hand, use the method of characteristics plus a first-collision approximation to obtain predictions for the

Delta J. Sci. 17 (3) 1993  
 M. Abdel-Aty Mohmoud

deviations of flow from the known free-molecular behaviour predicted by Johnson [3], [4].

Finally, Johnson [4] and stopford [5] present a crude approximation to the flow for very strong forces and for gases containing no more than a small but appreciable continuum region.

Here the Liu-Lees [9] approximation is modified so as to apply both for zero and nonzero external force. The method is then used to obtain predictions for gas flow and shear stress. These results apply to steady Couette flow at low Mach number, for any constant magnetic field, and arbitrary reflection coefficient and degree of rarefaction.

Plane steady Couette flow at low Mach number is studied by Johnson and Stopford [10] in the presence of an external force field. Force strengths may be so great that the gas is highly rarefied in one region of the flow but a continuum nearby.

In the present work we consider a steady Couette flow of charged particles between two infinite parallel plates moving in the xy plane opposite to each other in x-direction with velocity  $(\pm \frac{W}{2})$  under the influence of a constant magnetic field  $(0, 0, mB_0)$ . The whole system is assumed to be at constant temperature  $T_0$ . We consider also the particles reflected from the two plates with arbitrary reflection coefficients.

## 2. BASIC EQUATION OF THE RAREFIED GAS

The Boltzmann equation may be written in the following manner ( Cercignani 1975 ) [11]

Delta J. Sci. 17 (3) 1993  
STEADY COUETTE Flow

$$(\mathbf{v} \cdot \nabla + \frac{\mathbf{F}}{m} \cdot \nabla_{\mathbf{v}}) f(\mathbf{r}, \mathbf{v}) = J(f), \quad (1)$$

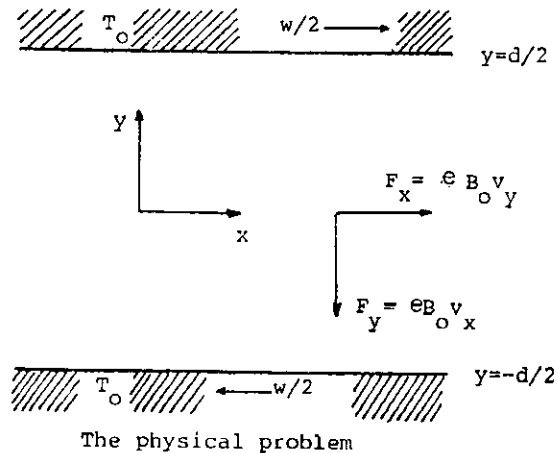
where  $f(\mathbf{r}, \mathbf{v})$  is the velocity distribution function,  $m$  the mass of a gas atom,  $\mathbf{F}$  the force acting on a gas atom and  $J(f)$  the non-linear collision term.

Now we reduce our problem to a plane symmetry in which the gas is confined between two infinite parallel plates at  $y = \pm d/2$  which are travelling in the  $x$ -direction with constant velocities  $\pm \frac{w}{2}$ . We also set the forces term  $F_x = eB_0 V_y$  and

$F_y = -eB_0 V_x$ ,  $F_z = 0$  Equation (1) becomes :

$$v_x \frac{\partial f}{\partial x} + v_y \frac{\partial f}{\partial y} + \frac{e}{m} B_0 v_y \frac{\partial f}{\partial v_x} - \frac{e}{m} B_0 v_x \frac{\partial f}{\partial v_y} = \frac{\sqrt{2RT_0}}{L} (f^{(0)} - f), \quad (2)$$

where  $L$  is the mean free path,  $f^{(0)}$  local Maxwellian distribution and  $T_0$  is the physical temperature.



Replacing the velocity and distance variables in eqn. (2) such that :

$$C_x = \sqrt{\alpha_o} v_x, \quad C_y = \sqrt{\alpha_o} v_y, \quad y = \bar{y}d, \quad x = \bar{x}d,$$

where  $d$  is the plate spacing, we find after dropping the bar over dimensionless quantities :

$$C_x \frac{\partial f}{\partial x} + C_y \frac{\partial f}{\partial y} + MC_y \frac{\partial f}{\partial C_x} - MC_x \frac{\partial f}{\partial C_y} = \delta (f^{(o)} - f) \quad (4)$$

where  $M = deB_o \sqrt{\alpha_o} / m$  magnetic Mach number,

$$\delta = \frac{d\sqrt{m}}{L} \quad \text{degree of rarefaction,}$$

$$\text{and } \alpha_o = \frac{m}{2RT_o}.$$

The reason for choosing this form of  $F$  is that it leads, in a locally Maxwellian gas, to a gas density of the form

$$n_{\pm}(x, y) = n_o(x) e^{-M^2(1-y^2)}. \quad (5)$$

The assumption of gentle flow allows one to linearize the Boltzmann equation for the molecular number-density distribution function  $f$  about the zero-shear distribution function  $f^{(o)}$ , such that:

$$f^{(o)} = n_o(x) e^{-M^2(1-y^2)} (\alpha_o/\pi)^{3/2} e^{-c^2}. \quad (6)$$

The solution of equation (4) is:

$$f = f_+ \theta(C_y - My) + f_- \theta(My - C_y), \quad (7)$$

where  $\theta$  is the Heaviside step function, and  $f_{\pm}$  are chosen to be Maxwellian distributions.

$$f_{\pm} = n_{\pm} (\alpha_{\pm}/\pi)^{3/2} e^{-(c-V_{\pm})^2}, \quad \alpha_{\pm} = \alpha_o. \quad (8)$$

Here  $V_{\pm}$  is  $y$ -dependent parameter determined by the requirement that  $f$  satisfy a suitable number of (lowest-order) moment of the governing equation. The predictions of this

Delta J. Sci. 17 (3) 1993  
STEADY COUETTE Flow

method for plane Couette flow in the absence of external force are discussed elsewhere [11], [1] and [12].

The solution (7) is discontinuous, not at  $C_y=0$ , but at  $C_y=MY$ . It is therefore necessary to approximate  $f$  by the above distribution function.

The further approximation of linearizing about the zero-shear solution (6), so that  $f_{\pm}$  may be written as:

$$f_{\pm} = f^{(0)} |1 + 2C_x V_{x\pm}(y)| \quad (9)$$

or

$$f_{\pm} = f^{(0)} |1 + 2C_x (V_{x+}(y) \theta(C_y - My) + V_{x-}(y) \theta(My - C_y))|.$$

In our problem, one wants to predict the flow velocity  $V$  and pressure deviator (shear stress)  $P_{xy}$ , where

$$V = \frac{1}{n} \int v_x f dv, \quad (10)$$

$$P_{xy} = \int m v_x v_y f dv, \quad (11)$$

Equations (10), (11) and (9) give:

$$V = (1/2\sqrt{\alpha_0}) \{ (V_{x+} + V_{x-}) + (V_{x-} - V_{x+}) \operatorname{erf}(MY) \},$$

and

$$P_{xy} = P_0 \left( \frac{n_0(x) e^{-M^2}}{\sqrt{\pi}} \right) (V_{x+} - V_{x-}),$$

where  $P_0$  is the scalar pressure taken at either wall, and  $\operatorname{erf}$  denotes the usual error function.

Predictions for  $V_{x\pm}$  are obtained by using equation (2) after transform to moments equation in the form:

Delta J. Sci. 17 (3) 1993  
M. Abdel-Aty Mohmoud

$$\begin{aligned} & \frac{\partial}{\partial x} \int C_x \phi_i f d\vec{c} + \frac{\partial}{\partial y} \int C_y \phi_i f d\vec{c} - M \int C_y f \frac{\partial \phi_i}{\partial C_x} d\vec{c} \\ & + M \int C_x f \frac{\partial \phi_i}{\partial C_y} d\vec{c} = \delta \int \phi_i (f^{(0)} - f) d\vec{c}. \end{aligned} \quad (12)$$

If we put  $\phi_1 = C_x$ ,  $\phi_2 = C_x C_y$  respectively, we get two moment equations:

$$\begin{aligned} & \frac{1}{n_o(x)} \frac{dn_o(x)}{dx} + \frac{e^{-M^2 y^2}}{\sqrt{\pi}} \frac{d}{dy} (V_{x^+} - V_{x^-}) = -\frac{2\delta}{\sqrt{\pi}} | (V_{x^+} + V_{x^-}) + \\ & + (V_{x^-} - V_{x^+}) \operatorname{erf}(My) |, \end{aligned} \quad (13)$$

and

$$\begin{aligned} & \frac{d}{dy} e^{M^2 y^2} \left| \frac{\sqrt{\pi}}{4} (V_{x^+} + V_{x^-}) + \frac{\sqrt{\pi}}{4} \operatorname{erf}(My) - \frac{My}{2} e^{-M^2 y^2} (V_{x^-} - V_{x^+}) \right. \\ & \left. = \frac{\delta}{2} (V_{x^-} - V_{x^+}) \right. \end{aligned} \quad (14)$$

By using separation method, we get:

$$\frac{1}{n_o(x)} \frac{dn_o(x)}{dx} = k \rightarrow n_o(x) = D e^{kx}, \quad (15)$$

where K and D are constants,

$$\frac{e^{-M^2 y^2}}{2\delta} \frac{d}{dy} (V_{x^-} - V_{x^+}) = | (V_{x^+} + V_{x^-}) + (V_{x^-} - V_{x^+}) \operatorname{erf}(My) | + \frac{k}{2\delta} \sqrt{\pi}, \quad (16)$$

and

$$\begin{aligned} & \frac{\sqrt{\pi}}{4} \frac{d}{dy} e^{M^2 y^2} \left| (V_{x^+} + V_{x^-}) + (V_{x^-} - V_{x^+}) \operatorname{erf}(My) \right| - \\ & - \frac{d}{dy} \left| \frac{My}{2} (V_{x^-} - V_{x^+}) \right|. \end{aligned} \quad (17)$$

The last two equations may be reduced to :

$$\frac{d^2(V_{x^-} - V_{x^+})}{dy^2} - Ay \frac{d}{dy} (V_{x^+} + V_{x^-}) (V_{x^-} - V_{x^+}) = cye^{M^2 y^2}, \quad (18)$$

$$\frac{d}{dy} (V_{x^-} - V_{x^+}) = 2\delta e^{M^2 y^2} [(V_{x^+} + V_{x^-}) + (V_{x^-} - V_{x^+}) \operatorname{erf}(My)] + K\sqrt{\pi} e^{M^2 y^2}, \quad (19)$$

$$\text{where } A = \frac{4M\delta}{\sqrt{\pi}}, \quad B = \frac{4\delta^2}{\sqrt{\pi}} \quad \text{and} \quad C = 2M^2 k\sqrt{\pi}.$$

### 3. BOUNDARY CONDITIONS

The half-range nature of the expansion is away of incorporating into a polynomial expansion the discontinuity of the distribution function in velocity space which ordinary occurs at the boundaries  $y = \pm 1/2$ . The boundary condition for the single component of charged particles is different but physically simpler than those suggested (Shidovisky 1967) [1] for ordinary two component fully ionized charged particles in that fraction  $\epsilon$  of the electrons hitting a surface leave that surface with Maxwellian Velocity distribution characteristic of that surfaces velocity and temperature. The remaining  $(1-\epsilon)$  are assumed to undergo specular reflection.

The boundary conditions are:

$$\left. \begin{aligned} \theta(My - C_y) f_- &= (1-\epsilon) \theta(C_y - My) f_+ + \epsilon \theta(My - C_y) f_{S^-} \\ \theta(C_y - My) f_+ &= (1-\epsilon) \theta(My - C_y) f_- + \epsilon \theta(C_y - My) f_{S^+} \end{aligned} \right\} \quad (20)$$

where

$$f_{S^+} = n_0(x) e^{-M^2(1-y^2)} (\alpha_0/\pi)^{3/2} e^{-|C+(M'/2)\underline{i}|^2},$$

$$f_{S^-} = n_0(x) e^{-M^2(1-y^2)} (\alpha_0/\pi)^{3/2} e^{-|C-(M'/2)\underline{i}|^2},$$

where  $\underline{i}$  is a unit vector in the +x direction and  $M' = \sqrt{\alpha_0} w$  is the flow Mach number, where  $M' \ll 1$ .



Multiplying both sides in equations (20) by  $c_x$  then:

$$V_{x^-} (1/2) \{1 + \operatorname{erf}(M/2)\} = (1 - \epsilon) V_{x^+} (-1/2) \{1 - \operatorname{erf}(-M/2)\} + \frac{\epsilon M'}{2} \{1 + \operatorname{erf}(M/2)\},$$

$$V_{x^+} (-1/2) \{1 - \operatorname{erf}(M/2)\} = (1 - \epsilon) V_{x^-} (+1/2) \{1 + \operatorname{erf} M/2)\} - \frac{\epsilon M'}{2} \{1 - \operatorname{erf}(-M/2)\},$$

By solving the last two equations together we get:

$$\left. \begin{aligned} V_{x^+} (-1/2) &= -\frac{\epsilon}{(2-\epsilon)} \frac{M'}{2}, \\ V_{x^-} (1/2) &= \frac{\epsilon}{(2-\epsilon)} \frac{M'}{2}, \end{aligned} \right\} \quad (21)$$

#### 4. METHOD OF SOLUTION

Solving eq.s (18) and (19) under the boundary conditions (21), by using the series method for  $M^5 \ll 1$ , we get:

$$\begin{aligned} V_{x^-} = & \frac{e^{-M^2 y^2}}{4\delta} \left| a_0 \left( Dy + \frac{E}{6} y^3 + \frac{G}{120} y^5 \right) + a_1 \left( 1 + \frac{I}{2} y^2 + \frac{J}{24} y^4 \right) - \right. \\ & \left. - \frac{(6H+C)}{I} - 3Hy^2 - 5Fy^4 \right| + 1/2 (1 - \operatorname{erf} My) \left| a_0 \left( 1 + \frac{D}{2} y^2 + \frac{E}{24} y^4 + \right. \right. \\ & \left. \left. + \frac{G}{720} y^6 \right) + a_1 \left( y + \frac{I}{6} y^3 + \frac{J}{120} y^5 \right) - \frac{(6H+C)}{I} y - Hy^3 - Fy^5 \right| - k \frac{\sqrt{\pi}}{4\delta} \end{aligned} \quad (22)$$

$$\begin{aligned} V_{x^+} = & \frac{e^{-M^2 y^2}}{4\delta} \left| a_0 \left( Dy + \frac{E}{6} y^3 + \frac{G}{120} y^5 \right) + a_1 \left( 1 + \frac{I}{2} y^2 + \frac{J}{24} y^4 \right) - \right. \\ & \left. - \frac{(6H+C)}{I} - 3Hy^2 - 5Fy^4 \right| - 1/2 (1 + \operatorname{erf} My) \left| a_0 \left( 1 + \frac{D}{2} y^2 + \frac{E}{24} y^4 + \right. \right. \end{aligned}$$

Delta J. Sci. 17 (3) 1993  
STEADY COUETTE Flow

$$+\frac{G}{720}y^6)+a_1\left(y+\frac{I}{6}y^3+\frac{J}{120}y^5\right)-\frac{(6H+C)}{I}y-Hy^3-Fy^5\left|-\frac{k\sqrt{\pi}}{4\delta}\right. \quad (23)$$

where

$$F=\frac{CM^4}{2(6A+B)}, \quad H=\frac{CM^2(6A+B)+10CM^4}{(4A+B)(6A+B)},$$

$$D=(A+B), \quad E=(A+B)(3A+B),$$

$$G=(A+B)(3A+B)(5A+B), \quad I=(2A+B), \quad J=(2A+B)(4A+B),$$

$$a_0=eM/(2-e)/\frac{e^{-M^2/4}}{\delta}\{(D/4+E/96+G/120 \times 64)$$

$$+(1-\operatorname{erf} M/2)(1+D/8+E/24 \times 16+G/720 \times 64)\},$$

and

$$a_1=\frac{e^{-M^2/4}(3H/4+5F/16+(6H+C)/I)}{e^{-M^2/4}(1+I/8+J/24 \times 16)+\delta(1-\operatorname{erf} M/2)(1+I/24+J/120 \times 16)}$$

$$+\frac{\delta(1-\operatorname{erf} M/2)(H/4+F/16+(6H+C)/I)+K\sqrt{\pi}}{e^{-M^2/4}(1+I/8+J/24 \times 16)+\delta(1-\operatorname{erf} M/2)(1+I/24+J/120 \times 16)}$$

## 5. DISCUSSION OF RESULTS

From the above results we get the macroscopic velocity  $V$  of the flow in the form:

$$V=\frac{e^{-M^2y^2}}{4\delta\sqrt{a_0}}\left\{a_0\left(Dy+\frac{E}{6}y^3+\frac{G}{120}y^5\right)+a_1\left(1+\frac{I}{2}y^2+\frac{J}{24}y^4\right)\right. \quad (24)$$

$$\left.-\frac{(6H+C)}{I}-3Hy^2-5Fy^4-k\sqrt{\pi}e^{M^2y^2}\right\},$$

the shear stress  $P_{xy}$  in the form :

$$P_{xy} = -P_o |n_o(x) \frac{e^{-M^2}}{\sqrt{\pi}} \{ |a_o(1 + \frac{D}{2}y^2 + \frac{E}{24}y^4 + \frac{G}{720}y^6) + a_1(y + \frac{I}{6}y^3 + \frac{G}{120}y^5) + \frac{(6H+C)}{I}y - Hy^3 - Fy^5 | \}. \quad (25)$$

and one may wish to describe the relation between shear stress (25) and flow velocity (24) in terms of phenomenological coefficient of viscosity  $\mu$ , specified so as to be meaningful even in the Knudsen regime, In the present problem, a suitable definition of the viscosity is:

$$\begin{aligned} \mu &= -P_{xy} \left( \frac{dV}{dy} \right)^{-1}, \text{ i.e.,} \\ \mu &= P_o \left| \frac{n_o(x) e^{-M^2}}{\sqrt{\pi}} \{ |a_o(1 + \frac{D}{2}y^2 + \frac{E}{24}y^4 + \frac{G}{720}y^6) + a_1(y + \frac{I}{6}y^3 + \frac{J}{120}y^5) - \frac{(6H+C)}{I}y - Hy^3 - Fy^5 | \} \right. \\ &\quad \left. \frac{e^{-M^2y^2}}{4\delta\sqrt{\alpha_o}} \{ |a_o(D + \frac{E}{2}y^2 + \frac{G}{24}y^4) a_1(Iy + \frac{J}{8}y^3) - 6Hy - 20Fy^3 | \right. \\ &\quad \left. - 2M^2y | a_o(Dy + \frac{E}{6}y^3 + \frac{G}{120}y^5) + a_1(1 + \frac{I}{2}y^2 + \frac{J}{24}y^4) \right. \\ &\quad \left. - \frac{(6H+C)}{I} - 3Hy^2 - 5Fy^4 \} \right| \quad (26) \end{aligned}$$

The situation here is a matter of proper condition that are imposed at the boundaries. We shall discuss the dependence of the flow velocity function, shear stress and coefficient of viscosity on magnetic field, degree of rarefaction and reflection coefficient.

(i) For the absence of a magnetic Mach number  $M=0$  and for

Delta J. Sci. 17 (3) 1993  
STEADY COUETTE Flow

arbitrary degree of rarefaction, the nondimensional velocity, shear stress and viscosity coefficient are given by:

$$V = \frac{1}{4\delta\sqrt{\alpha_0}} \left| a'_0 \left( By + \frac{B^2}{6} y^3 + \frac{B^3}{120} y^5 \right) + a'_1 \left( 1 + \frac{B}{2} y^2 + \frac{B^2}{24} y^4 \right) - k\sqrt{\pi} \right|,$$

$$P_{xy} = -P_0 \left| \frac{n_0(x)}{\sqrt{\pi}} \right| \left| a'_0 \left( 1 + \frac{B}{2} y^2 + \frac{B^2}{24} y^4 + \frac{B^3}{720} y^6 \right) + a'_1 \left( y + \frac{B}{6} y^3 + \frac{B^2}{120} y^5 \right) \right|$$

and

$$\mu = P_0 \left| \frac{n_0(x)}{\sqrt{\pi}} \right| \left| a'_0 \left( 1 + \frac{B}{2} y^2 + \frac{B^2}{24} y^4 + \frac{B^3}{720} y^6 \right) + a'_1 \left( y + \frac{B}{6} y^3 + \frac{B^2}{120} y^5 \right) \right| /$$

$$/ \frac{1}{4\delta\sqrt{\alpha_0}} \left| a'_0 \left( B + \frac{B^2}{2} y^2 + \frac{B^3}{24} y^4 \right) + a'_1 \left( By + \frac{B^2}{6} y^3 \right) \right|,$$

where

$$a'_0 = \frac{eM' / (2-\epsilon)}{\frac{1}{\delta} \left( \frac{B}{4} + \frac{B^2}{96} + \frac{B^3}{120 \times 64} \right) + \left( 1 + \frac{B}{8} + \frac{B^2}{24 \times 16} + \frac{B^3}{720 \times 64} \right)}$$

and

$$a'_1 = \frac{k\sqrt{\pi}}{(1 + B/8 + B^2/24 \times 16) + \delta(1 + B/24 + B^2/120 \times 16)}$$

(ii) For the very dilute gas,  $K_n \rightarrow \infty$  ( $\delta \rightarrow 0$ ) and in the absence of a magnetic Mach number, the nondimensional velocity is given by:

$$\frac{\partial V_{\pm}(y)}{\partial y} = 0 \Rightarrow V = \text{constant} = v_0.$$

(iii) For the very dense gas, the continuous gas,  $K_n \rightarrow 0$  ( $\delta \rightarrow \infty$ ) and in the absence of a magnetic Mach number, the flow

Delta J. Sci. 17 (3) 1993  
 M. Abdel-Aty Mohmoud

velocity rate in either half space  $y \leq 0$

$$\frac{\partial V_{\pm}(y)}{\partial y} = \theta(y) - \theta(-y) \quad , \text{ by integrating}$$

$$V(y) = y + v_0.$$

This shows that the flow velocity is linear.

Cases(ii),(iii) agree respectively with the zeroth approximation—the Kundsen collisionless gas—and the first approximation—the Navier Stokes equations—of the hydrodynamic equations for hard sphere model in a continuous gas derived from Boltzmann equation.

## 6. CONCLUSIONS

From evaluation of the above formula (24) and (25) we obtain the following results shown in figures |1-5|, for the parameters  $\alpha_0 = 0.17$  ,  $k = -.001$  ,  $P_0 = 100$  Torr and  $n_0(x) = 2.68699 \times 10^{19} \text{ cm}^{-3}$ .

- (i) The flow velocity of the gas increases when the distance  $y$  increase for constant degree of rarefaction  $\delta$  , reflection coefficient  $\epsilon$  and a magnetic Mach number  $M$ , and it decreases as the degree of rarefaction  $\delta$  increase for constant  $y$ ,  $M$  and  $\epsilon$  as can be seen from figure |1|.
- (ii) The magnitude of flow velocity increases as the increase of reflection coefficient  $\epsilon$  for constant  $y$  and  $M$  , and it increases as  $y$  increases for constant  $\delta$  and  $M$  as shown in figure |2|.
- (iii) The flow velocity  $V$  increases when the magnetic Mach number  $M$  decreases for constant  $\delta, \epsilon$  and  $y$ , and  $V$  decreases when it reaches to the two surfaces for constant Mach number  $M$  and  $\epsilon$  , as it seen from figure |3|.

- (iv) The magnitude of pressure  $P_{xy}$  increases as the increase of a magnetic Mach number  $M$  for constant  $y$ ,  $\epsilon$  and  $\delta$ , and it increases with the increase of  $y$  for constant  $M$ ,  $\epsilon$  and  $\delta$ , as can be seen from figure |4|.
- (v) The pressure  $P_{xy}$  increases when the reflection coefficient  $\epsilon$  increases for constant  $y$ ,  $M$  and  $\delta$ , and  $P_{xy}$  increases when it reaches to the two surfaces for constant  $M$ ,  $\epsilon$  and  $\delta$  as can be seen from figure |5|.

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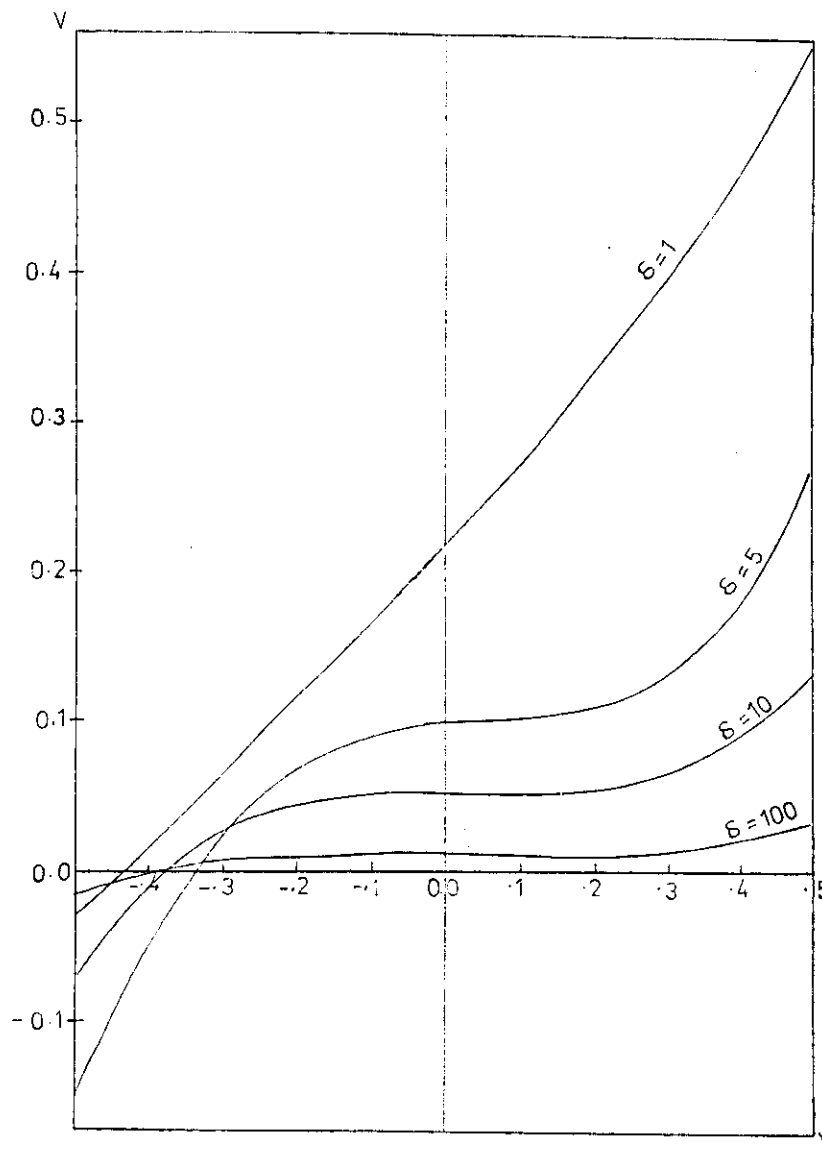


Fig.1: Variation of flow velocity  $V$  with  $\gamma$  for different values of degree of rarefaction  $\delta$ ,  $M=0.1$ ,  $\varepsilon=0.5$  and  $\tilde{M}=0.05$ .



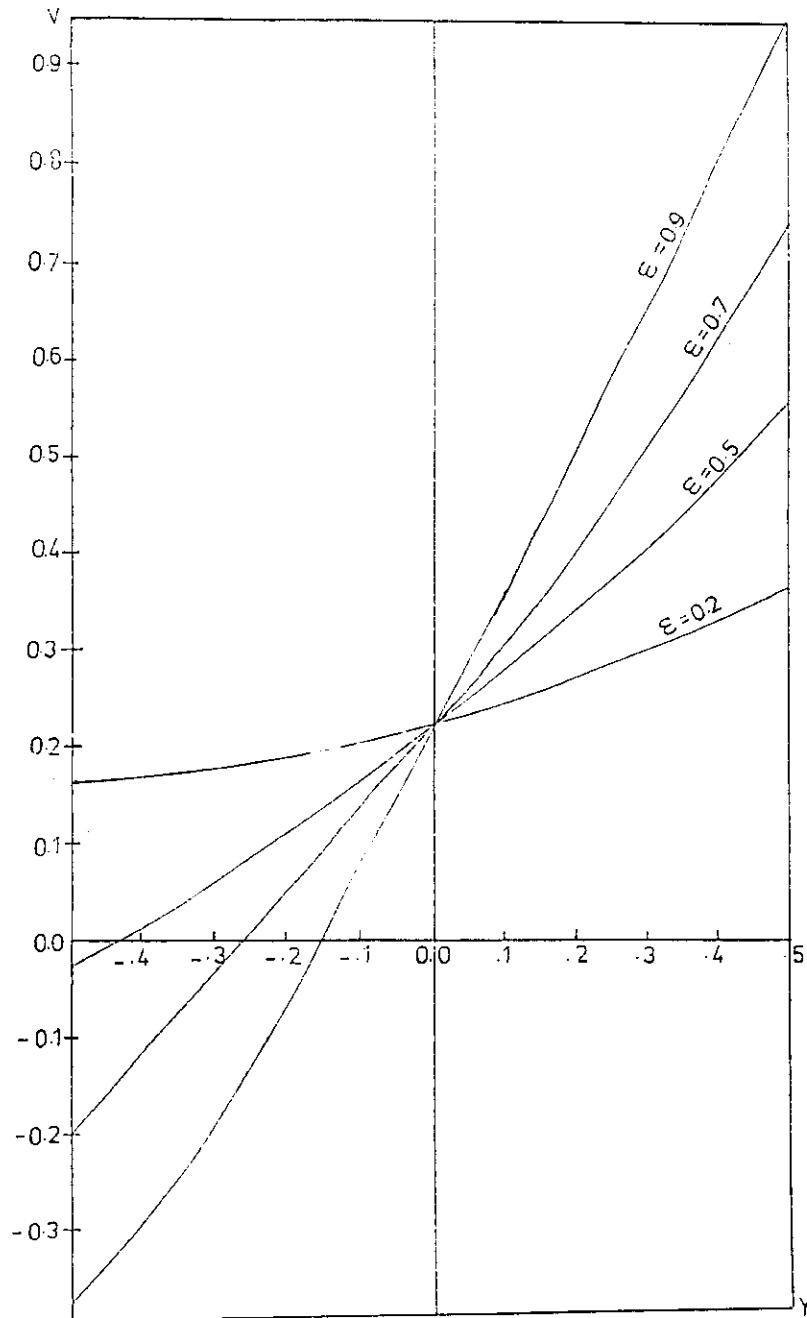


Fig.2 : Variation of V with Y for different values of reflection coefficient  $E$ ,  $M=0.1$ ,  $\delta=1$  and  $\tilde{M}=0.05$ .

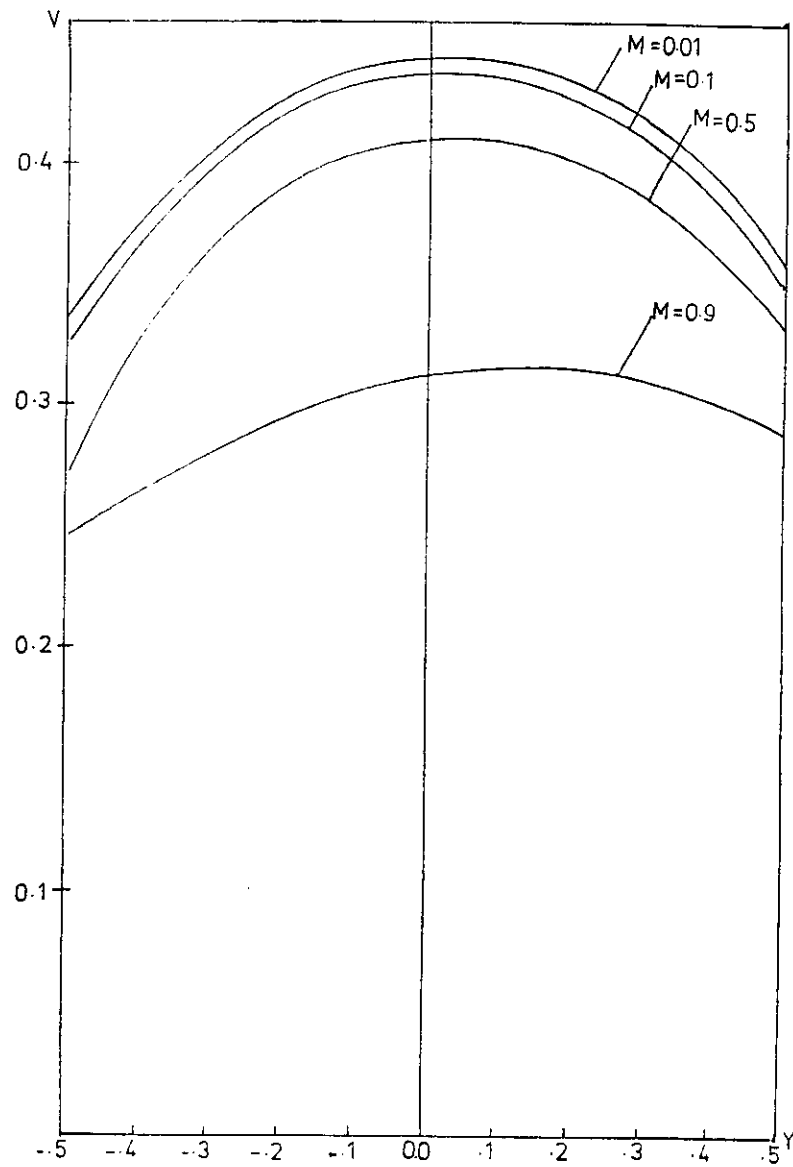


Fig.3 : Variation of V with Y for different values of

magnetic Mach number M,  $\delta = 1$ ,  $\bar{M} = 0.05$  and  $\epsilon = 0.5$ .

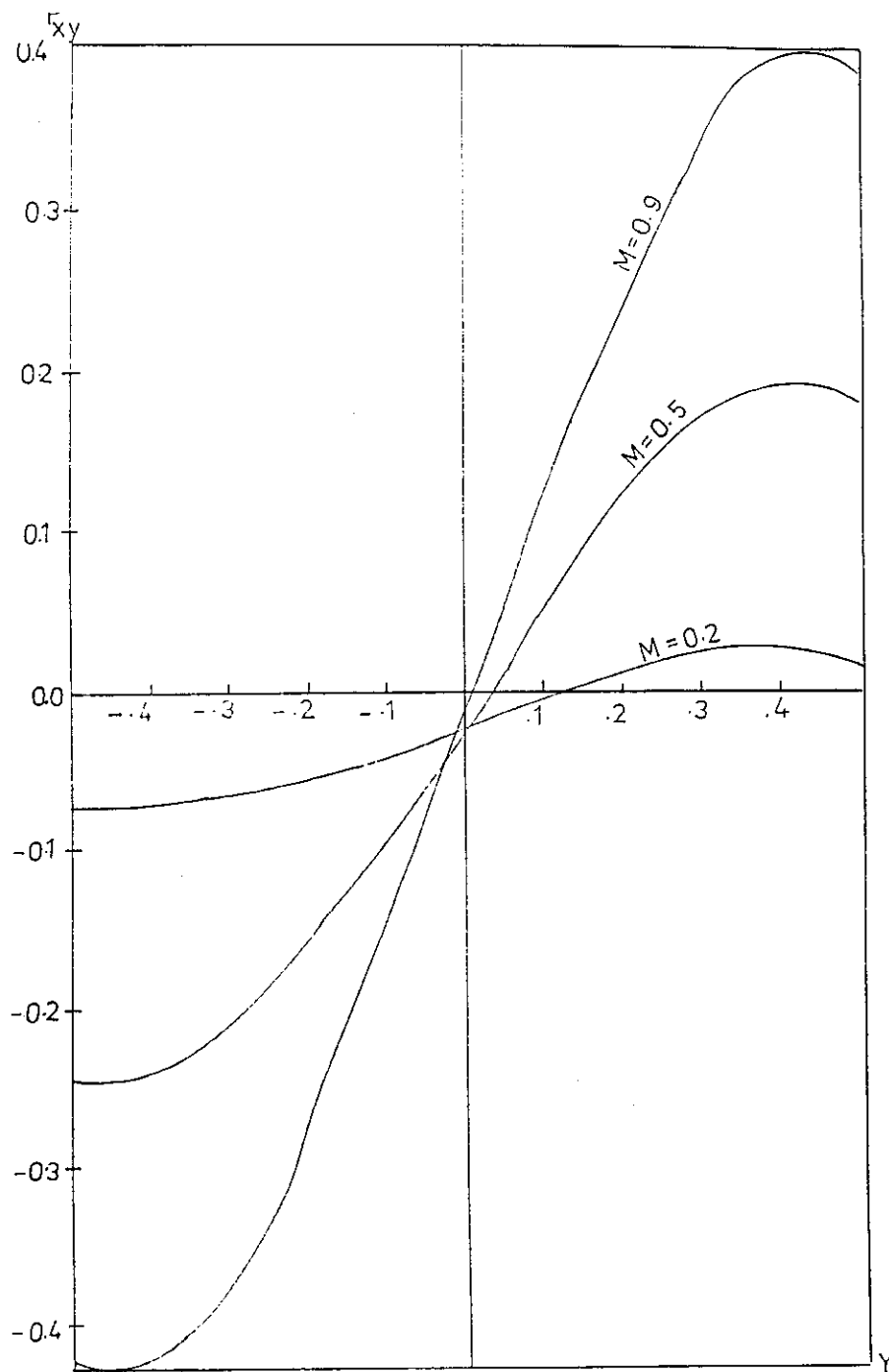
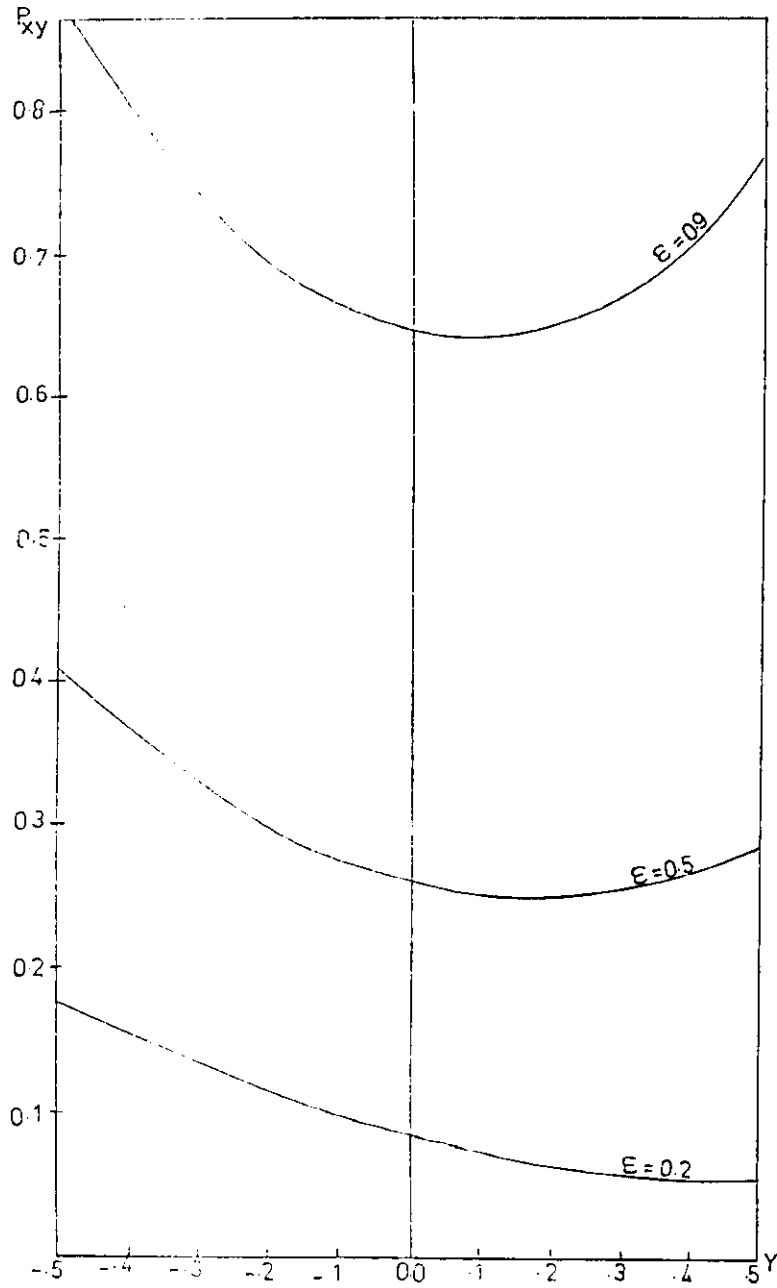


Fig.4 : Variation of pressure  $P_{xy}$  with  $Y$  for different values of  $M$ ,  $\delta = 1$ ,  $\epsilon = 0.5$  and  $\mu = 0.001$ .

Fig.5. Variation of  $P_{xy}$  with  $Y$  for different values of  $E$ . $G = 1$ ,  $M = 0.01$  and  $\tilde{M} = 0.001$

انسياب كويت المنتظم لغاز مخلخل مكون من جسيمات  
مشحونة يتحرك بين سطحين في مجال مغناطيسي  
ثابت مع وجود معاملات انعكاس اختيارية عند السطحين

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درسنا في هذا البحث مسألة انسياب كويت المنتظم عند عدد مساح المنخفض في وجود مجال مغناطيسي ثابت ، مع اعتبار أن الجسيمات المشحونة تنعكس عند السطحين العلوي والسفلي بمعاملات انعكاس اختيارية ، أيضا اعتبرنا أن كثافة الغاز الموجود بين اللوحين تعتمد على الاحداثى السيني في اتجاه حركة السطحين .  
وقد أوجد لدينا الحل التقريبي لمعادلة بولتزمان المعقدة استنتاج تعبير تحليلي مبسط لكل من سرعة الانسياب ، قوة القص ، وأيضا معامل اللزوجة .  
هذه الاستنتاجات أعطتنا نتائج صحيحة في كل من الغازات المخلخلة المتصلة والمحدودة ، وتظهر أسباب التوافق مع الاستنتاجات الدقيقة الأخرى في الغازات محكمة الحدود التي سبق ايجادها .  
وسوف نهمل للمجال الكهربائي الناشئ من توزيع الجسيمات المشحونة وأيضا المجال المغناطيسي الناشئ من حركة حجم الغازات المشحونة .

