

STEADY COUETTE FLOW OF A RAREFIED GAS  
CONSISTING OF CHARGED PARTICLES INSIDE POROUS  
WALLS WITH ARBITRARY REFLECTION COEFFICIENT

BY

Mahmoud, M. A.

Department of Math., Faculty of Science,  
Zagazig University, Benha 13518,

Egypt

Received:24-8-1993

ABSTRACT

Plane steady couette flow problem of rarefied gas law at Mach number is studied in the presence of constant magnetic field and porosity. The reflection coefficient of the upper and lower walls are considered different. The B.G.K. kinetic model has been solved using the moment method with two sided distribution function. The dependence of flow velocity, slip velocity, shear stress and coefficient of viscosity on reflection coefficient is obtained. We shall neglect electric fields arising from the distribution of charged particles and induced magnetic fields.

INTRODUCTION

The flow between two parallel infinite plates or concentric cylinders in relative tangential motion is called couette flow problem. Actually, one of the motives for studying the couette flow is the usefulness in studying boundary layer: it is sufficiently similar and considerably easier to solve. In the last few years, many investigators have been succeeded in obtaining approximate

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solutions for the plane couette flow of natural rarefied gas, which is suitable for any knudsen number.

The theory of interaction between gases and solid surfaces is far from being in final stage. Many authors such as (Shidlovskiy 1967), (Kogan 1969), (Chapman & Cowling 1970), (Khidr, 1970), (Cercignani 1975), (Hady 1976), (Johnson 1982), (Mahmoud 1985) made successful demonstrations of the gas flow using Boltzmann kinetic equation, specially in the study of Couette compressible flow between two solid parallel walls. Various models have been suggested, but the simplest of them takes the gas-surface interaction in the form of a tangential accommodation coefficient with respect to momentum. Different efforts were done for imposing the conditions at the boundaries, however they were not enough in describing a variety of phenomena that demand further predictions.

In the works of (Hady 1976), Mahmoud (1985) and Abourabia (1988) the equations of transfer were used to describe the problem of Couette flow in rarefied gases with porosity, they open a large area of study to follow this effect.

## 2. SETTING UP THE PROBLEM

In this paper steady Couette compressible flow between two parallel plates of the gas consists of charged particles moving under constant magnetic field whose components are  $(0, 0, B_0)$  and with porosity is discussed. We consider also that

the reflection coefficients of the upper and lower walls are different.

For simplicity it will be assumed that the flow is gentle i.e the Mach number sufficiently small such that

$$M^2 \ll 1 \quad , \quad M = \sqrt{\alpha_0} U,$$

where  $\alpha_0 = (2RT)^{-1}$ ,  $T$  is the constant temperature at the walls,  $U/2$  ( $-U/2$ ) is the velocity of the upper (lower) wall in the x-direction,  $d$  is the distance between the walls and  $y=d/2$  ( $-d/2$ ) is the equation of the upper (lower wall). As well known one of the important factors to investigate the effect of porosity is the gas flows out from the lower and upper walls with velocities  $V_1 = aU$  and  $V_2 = bU$ ,  $a < 0$ ,  $b > 0$ .

The relevant Boltzmann equation governing the present problem is

$$C_y \frac{\partial f}{\partial y} + \frac{e}{m} (B_0 C_y \frac{\partial f}{\partial C_x} - B_0 C_x \frac{\partial f}{\partial C_y}) = I(f) \quad (1)$$

where  $I(f)$  is the usual Boltzmann collision operator. Because shear is assumed to be weak, it is reasonable to linearize the number density distribution function  $f$  about the zero-shear maxwellian:

$$f_0 = n (\alpha_0 / \pi)^{3/2} \exp\{-\alpha_0 C^2\} \quad (2)$$

where  $n$  is the constant density number.

To obtain an approximate kinetic-theory solution to eq. (1) one may use the B.G.K equation with which a

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a molecule tends to relax to local equilibrium after a single collision. Thus

$$C_y \frac{\partial f}{\partial y} + \frac{e}{m} (B_0 C_y \frac{\partial f}{\partial C_x} - B_0 C_x \frac{\partial f}{\partial C_y}) = I(f) - \frac{\sqrt{2RT}}{L} (f_0 - f) \quad (3)$$

i.e. the processes of transfer of molecular quantities depend appreciably on the mean free path L.

Eq. (3) is nonlinear integrodifferential equation, there are many method for solving it. During the paper we look for a solution of eq. (3) by the approximation method of (Liu & lees 1961). The method starts by replacing the exact distribution function by a two stream Maxwellian. Then we use the moments method to solve eq. (3).

Multiplying both sides of equation (3) by  $\phi_1(\vec{C})$  and integrating over all values of  $\vec{C}$  we get for  $\phi_1(\vec{C})$

$$\begin{aligned} & \frac{d}{dy} (\overline{C_y \phi_1}) - \frac{eB_0}{m} \overline{C_y \frac{\partial \phi_1}{\partial C_x}} + \frac{eB_0}{m} \overline{C_x \frac{\partial \phi_1}{\partial C_y}} \\ & = \frac{\sqrt{2RT}}{L} \int \phi_1 (f_0 - f) d\vec{C} \end{aligned} \quad (4)$$

where, for any function of velocity  $Q_j(\vec{C})$  we have

$$\overline{Q_j} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} Q_j f^- dC_x dC_y dC_z + \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_0^{\infty} Q_j f^+ dC_x dC_y dC_z$$

and  $f^\pm$  is chosen to be:

$$\left. \begin{aligned} f^- &= n (\alpha_0 / \pi)^{3/2} (1 + V_1 C_y / RT) \exp -\alpha_0 [(C_x - V_{x1})^2 + C_y^2 + C_z^2] : C_y < 0 \\ f^+ &= n (\alpha_0 / \pi)^{3/2} (1 + V_2 C_y / RT) \exp -\alpha_0 [(C_x - V_{x2})^2 + C_y^2 + C_z^2] : C_y > 0 \end{aligned} \right\} (5)$$

Here  $V_{x1}, V_{x2}$  are y-dependent parameters determined by the requirement that  $f$  satisfies a suitable number of (lower order) moment of the governing equations.

In this paper, we are going to predict the flow velocity function, shear stress and coefficient of viscosity.

### 3. THE BOUNDARY CONDITIONS

The boundary conditions on each  $V_{x1}$  and  $V_{x2}$  are obtained from the assumption that a fraction  $\theta_1$  of molecules is re-emitted with a Maxwellian distribution,  $(1-\theta_1)$  is reflected specularly from the lower plate, that a fraction  $\theta_2$  of molecules is re-emitted with a Maxwellian distribution, and  $(1-\theta_2)$  is reflected specularly from the upper plate. Using functions (4), it is desired to integrate two equations of the form,

$$f^\pm = (1-\theta_{1,2}) f^\mp + \theta_{1,2} f_{s_{1,2}}$$

with respect to  $C_x$ , where

$$f_{s_{1,2}} = n(\alpha_0/\pi)^{3/2} [1 + V_{1,2} C_y / RT] \exp\{-\alpha_0 [(C_x \pm U/2)^2 + C_y^2 + C_z^2]\}$$

Hence we obtain at the lower wall in nondimensional form

$$V_{x1}(-1/2) = (1-\theta_2) S V_{x2}(1/2) - \theta_1/2 \quad (6)$$

and at the upper wall

$$V_{x2}(1/2) = (1-\theta_1) S^{-1} V_{x1}(-1/2) + \theta_2/2 \quad (7)$$

where  $y$  and both  $V_{x1,2}, V_{1,2}$  are nondimensionalized

with respect to  $d$  and  $U$  respectively.

The suction factor  $S$  is equal to

$$S = (1/2 + \gamma b/\pi) (1/2 - \gamma a/\pi)^{-1}$$

where  $\gamma = \frac{U}{2} \sqrt{\frac{2\pi}{RT}}$  is the same order of Mach number.

The solution of the system of equations (6), (7) are obtained in terms of the arbitrary coefficients  $\theta_1, \theta_2$  and  $S$  as follows;

$$V_{x1}(-1/2) = [\theta_2(1-\theta_2)S - \theta_1] [2(\theta_1 + \theta_2 - \theta_1\theta_2)]^{-1} \quad (8)$$

$$V_{x2}(1/2) = [\theta_2S - (1-\theta_1)\theta_1] [2S(\theta_1 + \theta_2 - \theta_1\theta_2)]^{-1} \quad (9)$$

#### 4. THE CHARACTERISTICS OF THE FLOW

In this section the determination of  $V_{x1}$  and  $V_{x2}$  are obtained by putting

$$\phi_1 = C_x, \quad \phi_2 = C_x C_y \text{ in eq. (4)}$$

when  $\phi_1 = C_x$  eq. (4) becomes in the following nondimensional form

$$\frac{d}{dy} [(V_{x2} - V_{x1}) + \gamma(aV_{x1} - bV_{x2})] = \gamma\beta(a-b) \quad (10)$$

when  $\phi_2 = C_x C_y$ , eq. (4) becomes in the nondimensional form

$$\begin{aligned} & \frac{d}{dy} [1/2(V_{x2} + V_{x1}) - \frac{\gamma}{\pi}(aV_{x1} + bV_{x2})] + \\ & + \frac{\beta\gamma^2}{\pi}(V_{x1}^2 + V_{x2}^2) - \frac{2\beta\gamma^3}{\pi^2}(aV_{x1} + bV_{x2}) + \frac{\gamma\beta}{\pi}(a+b) \\ & = -\delta [(V_{x2} - V_{x1}) + \gamma(aV_{x1} - bV_{x2})] \end{aligned} \quad (11)$$

where  $\beta = \frac{eB_0 d}{mU}$  is a constant related to magnetic field, and  $\delta = \frac{d}{L\sqrt{\kappa}}$  is the degree of rarefaction.

We shall solve eq.s (10) and (11) by using the method of small parameter. Considering  $\gamma$  sufficiently small such that  $\gamma^2 \ll 1$ , we seek the solution in the form

$$V_{xi} = V_{xi}^{(0)} + \gamma V_{xi}^{(1)}$$

On substituting these values in eq.s(10), (11) and equating the terms free of  $\gamma$  on both sides of these equations, we get two equations by solving it simultaneously using the boundary conditions, we obtain:

$$V_{x1}^{(0)} = -\delta C_1 y + C_3 - \frac{C_1}{2} \quad , \quad (12)$$

$$V_{x2}^{(0)} = -\delta C_1 y + C_3 + \frac{C_1}{2} \quad , \quad (13)$$

where

$$C_1 = \frac{(\theta_1^2 + \theta_2^2)}{8(1-\delta)(\theta_1 + \theta_2 - \theta_1\theta_2)} \quad , \quad (14)$$

$$C_3 = \frac{(\theta_2 - \theta_1)(1 - \theta_1 - \theta_2)}{16(\theta_1 + \theta_2 - \theta_1\theta_2)} \quad (15)$$

Then equating the coefficients of  $\gamma$  on both sides we have ;

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$$V_{x1}^{(1)} = -\frac{\delta}{2} (a-b) \beta y^2 - \left[ \frac{(a+b)}{\pi} (\beta + \delta C_1) + \delta C_2 + (\delta C_1 + \beta) \frac{(a-b)}{2} \right] y$$

$$+ \left[ \frac{2(a+b) + \pi(a-b)}{2\pi} \right] C_3 - \left[ \frac{2(a-b) + \pi(a+b)}{4\pi} \right] C_1 + C_4 - \frac{1}{2} C_2 \quad (16)$$

$$V_{x2}^{(1)} = -\frac{\delta}{2} (a-b) \beta y^2 + \left[ \frac{(a-b)}{2} (\delta C_1 + \beta) - \frac{(a+b)}{\pi} (\beta + \delta C_1) - \delta C_2 \right] y$$

$$+ \left[ \frac{2(a+b) - \pi(a-b)}{2\pi} \right] C_3 + \left[ \frac{\pi(a+b) - 2(a-b)}{4\pi} \right] C_1 + C_4 + \frac{1}{2} C_2 \quad (17)$$

where

$$C_4 = \left\{ (a-b) (\theta_1 - \theta_1^2 + \theta_2^2) \right\} \left[ 8\pi (\theta_1 + \theta_2 - \theta_1 \theta_2) \right]^{-1} - \left[ \frac{(a+b)}{\pi} \right] C_3$$

$$+ \left[ \beta (a-b) (\delta - 2) / 8 \right] + \left[ (a-b) (2 - \pi \delta) / 4\pi \right] C_1 \quad (18)$$

and

$$C_2 = \left\{ (a-b) (2\theta_2 - \theta_1 - \theta_2^2 + \theta_1^2) \right\} \left[ 4\pi (1 - \delta) (\theta_1 + \theta_2 - \theta_1 \theta_2) \right]^{-1}$$

$$+ \left[ \beta (a+b) / \pi (\delta - 1) \right] + \left[ (a+b) (2\delta + \pi) / 2\pi (\delta - 1) \right] C_1$$

$$- \left[ (a-b) / (\delta - 1) \right] C_3 \quad (19)$$

### 5. DISCUSSION OF THE RESULTS

From the above results we get the macroscopic velocity  $V$  of the flow form:

$$V = \frac{(\pi + 2\gamma a)}{2\pi} (-\delta C_1 y + C_3 - \frac{1}{2} C_1) + \frac{(\pi + 2\gamma b)}{2\pi} (-\delta C_1 y + C_3 + \frac{1}{2} C_1) +$$

$$+ \frac{\gamma}{2} [-\delta (a-b) \beta y^2 + (\delta C_1 + \beta) (a-b) y - 2\delta C_2 y + \frac{2(a+b)}{\pi} C_3$$

$$- \frac{(a-b)}{\pi} C_1 + 2C_4] \quad (20)$$

The normal velocity of the flow is

$$V_y = \frac{1}{2} (a+b) \quad (21)$$



The shear stress  $P_{xy}$  is in the form:

$$P_{xy} = \frac{1}{4} (a-b) C_1 + \frac{\gamma (a+b)}{2\pi} \{-\delta C_1 (a-b) y + (a-b) C_3 - \frac{C_1}{2} (a+b)\} \\ + \frac{(a-b)}{4} \gamma [(\delta C_1 + \beta) (a-b) y - (a-b) C_3 + \frac{(a+b)}{2} C_1 + C_2] \quad (22)$$

and the coefficient of viscosity is in the form:

$$\mu = \left\{ \frac{(a-b)}{4} C_1 + \frac{\gamma (a+b)}{2\pi} [-\delta C_1 (a-b) y + (a-b) C_3 - \frac{C_1}{2} (a+b)] \right. \\ \left. + \frac{(a-b) \gamma}{4} [(\delta C_1 + \beta) (a-b) y - (a-b) C_3 + \frac{(a+b)}{2} C_1 + C_2] \right\} / \\ \left\{ \frac{-\delta C_1}{2\pi} (\pi + 2\gamma a) - \frac{\delta C_1}{2\pi} (\pi + 2\gamma b) + \frac{\gamma}{2} [-2\delta\beta (a-b) y \right. \\ \left. + (\delta C_1 + \beta) (a-b) - 2\delta C_2] \right\} \quad (23)$$

The situation studied here is a matter of proper condition that are imposed at the boundaries. We shall discuss the dependence of the flow velocity function, shear and coefficient of viscosity on the normal velocity, degree of rarefaction, magnetic field and reflection coefficient. We have the following cases;

- (i) For non porous plates, for arbitrary degree of rarefaction  $\delta$ , the nondimensional velocity and shear stress are given by:

$$V = -\delta C_1 y + C_3$$

$$P_{xy} = 0$$

- (ii) For porous plates and free molecular flow  $K_n \rightarrow \infty$  (when the collision may be neglected  $\delta = 0$ ) the

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dimensionless slip velocity and shear stress  
 are given by:

$$V_s = 1/2 - \dot{V}(1/2)$$

$$= \frac{1}{2} - \frac{(\theta_2 - \theta_1)(2 - \theta_1 - \theta_2)}{4(\theta_1 + \theta_2 - \theta_1\theta_2)} + \frac{\gamma(a-b)(\theta_1^2 + \theta_2^2)}{4\pi(\theta_1 + \theta_2 - \theta_1\theta_2)} - \frac{\gamma(a+b)(8\theta_2 - 4\theta_1 - 5\theta_2^2 + \theta_1^2)}{4\pi(\theta_1 + \theta_2 - \theta_1\theta_2)},$$

$$P_{xy} = \left[ \frac{1}{4}(a-b) - \frac{(a-b)^2}{4\pi}\gamma + \frac{(a^2 - b^2)}{8}\gamma \right] \frac{(\theta_1^2 + \theta_2^2)}{2(\theta_1 + \theta_2 - \theta_1\theta_2)} + \left[ \frac{\gamma(a^2 - b^2)}{2\pi} - \frac{(a-b)^2\gamma}{4} \right] \frac{(\theta_2 - \theta_1)(2 - \theta_1 - \theta_2)}{4(\theta_1 + \theta_2 - \theta_1\theta_2)} + \frac{\beta(a-b)^2}{4}\gamma y + \frac{(a-b)\gamma}{4} \left[ \frac{(a+b)(\theta_1 - \theta_2 - \theta_1^2 + \theta_2^2)}{\pi(\theta_1 + \theta_2 - \theta_1\theta_2)} + \frac{\beta(a+b)}{\pi} - \frac{(a+b)(\theta_1^2 + \theta_2^2)}{2(\theta_1 + \theta_2 - \theta_1\theta_2)} + \frac{(a-b)(\theta_2 - \theta_1)(2 - \theta_1 - \theta_2)}{4(\theta_1 + \theta_2 - \theta_1\theta_2)} \right]$$

(iii) For porous plates, continuous gas,  $K_n \rightarrow 0$  (when  $\delta \rightarrow \infty$ ) and in the absence of magnetic field the non-dimensional slip velocity and shear stress are given by:

$$V_s = \frac{1}{2} - \dot{V}\left(\frac{1}{2}\right) = \frac{1}{2} - \frac{\gamma(a+b)(\theta_1 + \theta_2 - \theta_1^2 - \theta_2^2)}{\pi(\theta_1 + \theta_2 - \theta_1\theta_2)} - \frac{(\theta_2 - \theta_1)(2 - \theta_1 - \theta_2)}{4(\theta_1 + \theta_2 - \theta_1\theta_2)}$$

$$P_{xy} = \gamma \left[ \frac{(a^2 - b^2)}{2\pi} - \frac{(a-b)^2}{4} \right] \frac{(\theta_2 - \theta_1)(2 - \theta_1 - \theta_2)}{4(\theta_1 + \theta_2 - \theta_1\theta_2)}$$

(iv) For porous plates and for any  $\delta$ , the slip velocity at the upper plate is given by:

$$\begin{aligned}
 V_s = & \frac{1}{2} + \frac{(\pi+2\gamma a)}{4\pi} (\delta C_1 + C_1 - 2C_3) - \frac{(\pi+2\gamma b)}{4\pi} (-\delta C_1 + C_1 + 2C_3) + \\
 & + \frac{\gamma\delta\beta}{8} (a-b) - \frac{\gamma}{4} (\delta C_1 + \beta) (a-b) + \frac{\gamma\delta}{2} C_2 - \frac{\gamma(a-b)}{\pi} C_3 \\
 & + \frac{\gamma(a-b)}{2\pi} C_1 - \gamma C_4 \qquad (24)
 \end{aligned}$$

(v) In the absence of porosity and magnetic field we have two limiting cases arise:

(1) The very dilute gas  $K_n \rightarrow \infty (\delta \rightarrow 0)$  eq. (20) gives

$$V'(y) = V_0, \text{ where } V_0 \text{ is an arbitrary constant.}$$

(2) The very dense gas, the continuous gas,

$$K_n \rightarrow 0 (\delta \rightarrow \infty) \text{ gives from eq. (20) } V'(y) = y + V_0.$$

This shows that the flow velocity is linear. The cases (1), (2) agree respectively with the Zeroth-approximation-the knudsen collisionless gas-and the first approximation- the Navier stokes equations of the hydrodynamic equations for hard sphere model in a continuous gas derived from the Boltzmann equation.

## 6. CONCLUSIONS

The values of velocity, shear stress and slip velocity are calculated from formula (20), (22) and (24) when the parameters take the following values and for different value of  $\theta_1, \theta_2$ . The results are given in Figures (1-9), these results gives:

1. (i) If the reflection coefficient is the same

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$\theta_1 = \theta_2 = \theta$ , the slip velocity at the upper plate  $V_s(\frac{1}{2})$

increases as  $\theta$  increases for any degree of rarefaction  $\delta$ , and it increases with the increase of  $\delta$  for any constant  $\theta$ , as it may be shown in Fig.(1)

- (ii) From Fig.(2) we see that the slip velocity increases as the difference between the reflection coefficient of the two plates ( $\theta = \theta_1 - \theta_2$ ) increases for any constant  $\delta$ , and it increases with the increase with the increase of  $\delta$  for any constant  $\theta$ .
- (iii) The slip velocity increases as the increase of the ratio  $a/b$  for constant  $\delta$  and  $\theta$ , and it increases with the increase of  $\delta$  for constant  $a/b$  and  $\theta$ , as it may be shown in Fig.(3).
2. (i) If the reflection coefficient is the same ( $\theta_1 = \theta_2 = \theta$ ), the magnitude of shear stress  $P_{xy}$  increases as  $\theta$  increases for any constant  $y$ , and it decreases as  $y$  increases for any constant  $\theta$  [see Fig. 4].
- (ii) From Fig.(5), we see that the magnitude of shear stress increases as the increase of  $\delta$  for constant  $y$  and  $\theta$ , and it decreases with the increase of  $y$  from the lower plate for constant  $\delta$  and  $\theta$ .
- (iii) The magnitude of shear stress  $P_{xy}$  decreases as the increase of the ratio  $a/b$  for constant  $y$

and  $\theta$ , and it decreases with the increase of  $y$  for constant  $a/b$  and  $\theta$ , as it may be shown in Fig. (6).

- 3.(i) If the reflection coefficient is the same ( $\theta_1 = \theta_2 = \theta$ ), the magnitude of the flow velocity  $V'$  increases with the increase of  $\theta$  for constant  $y$ , and it decreases with the increase of  $y$  from the lower plate for constant  $\theta$ , as it may be shown in Fig.(7).
- (ii) From Fig. [8], we see that the magnitude of flow velocity increases as  $\delta$  increases for any constant  $y$  and it decreases as  $y$  increases for any constant  $\delta$  and  $\theta$ .
- (iii) The magnitude of flow velocity decreases as the increase of  $y$  for constant  $a/b$  and it increases with the increase of  $a/b$  for constant  $y$  and  $\theta$ , see Fig.(9).

FIGURE CAPTIONS:

Fig. (1)-The relation between the slip velocity  $V_s$  and the degree of rarefaction  $\delta$  for the same values of  $\theta_1, \theta_2$ , and  $\beta$  (constant related to the magnetic field)=0.001.

Fig. (2)-The relation between the slip velocity  $V_s$  and the degree of rarefaction  $\delta$  for different values of  $\theta_1, \theta_2$  (reflection Coefficients) and  $\beta=0.001$ .

Fig. (3)-The relation between the slip velocity  $V_s$  and  $\delta$  for different values of  $a/b$  (ratio of

suction),  $\beta=0.001$  and  $\theta_1=\theta_2=0.8^\circ$

Fig. (4)-The relation between the shear stress  $P_{xy}$  and the distance  $y$  for the same values of  $\theta_1, \theta_2, \delta=0.4$  and  $\beta=0.001$ .

Fig. (5)- The relation between the shear stress  $P_{xy}$  and the distance  $y$  for different values of  $\delta$ ,  $\beta=0.001$  and  $\theta_1=\theta_2=0.5$ .

Fig. (6)- The relation between the shear stress  $P_{xy}$  and the distance  $y$  for different values of  $a/b$ ,  $\delta=0.5$ ,  $\beta=0.001$  and  $\theta_1=\theta_2=0.5$ .

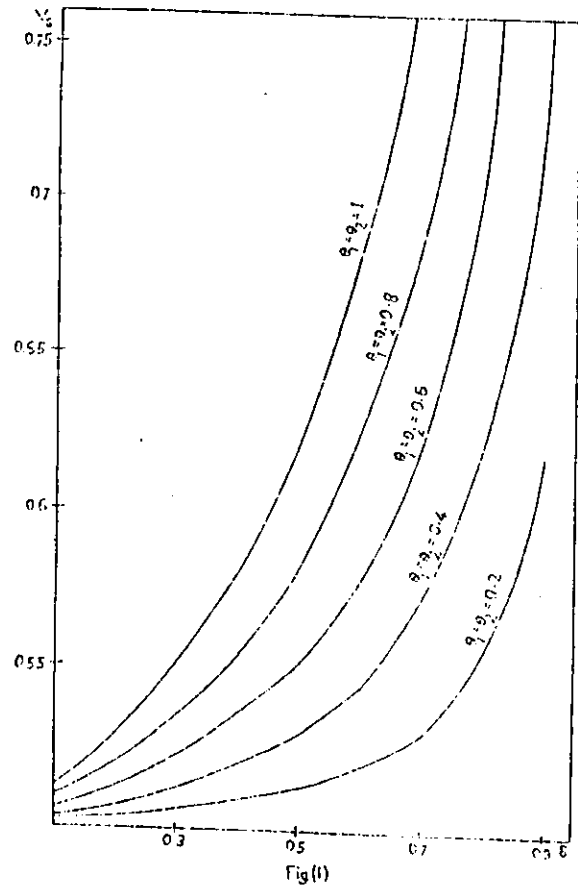
Fig. (7)- The relation between the flow velocity  $V'$  and distance  $y$  for the same values  $\theta_1, \theta_2, \beta=0.001$  and  $\delta=0.5$

Fig. (8)- The relation between the flow velocity  $V'$  and the distance  $y$  for different values of  $\delta, \beta=0.001$  and  $\theta_1=\theta_2=0.5$ .

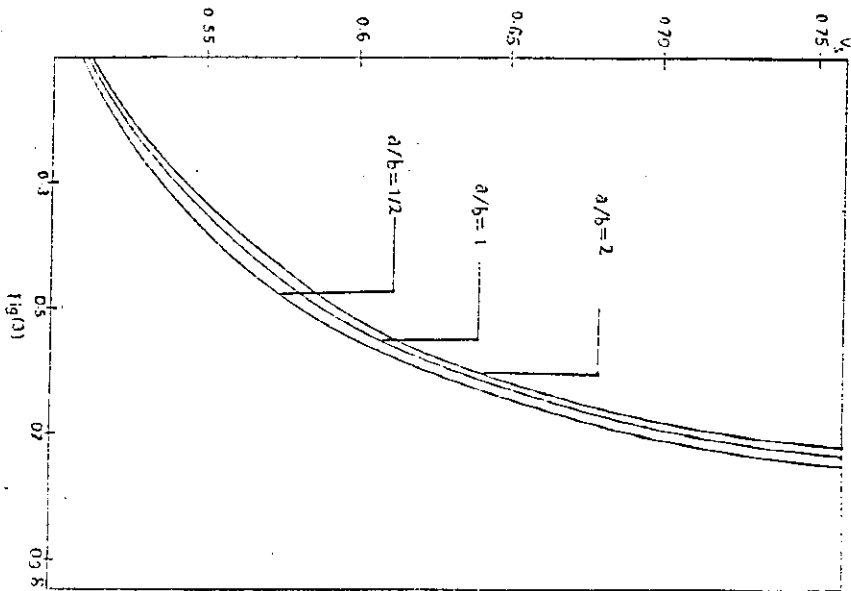
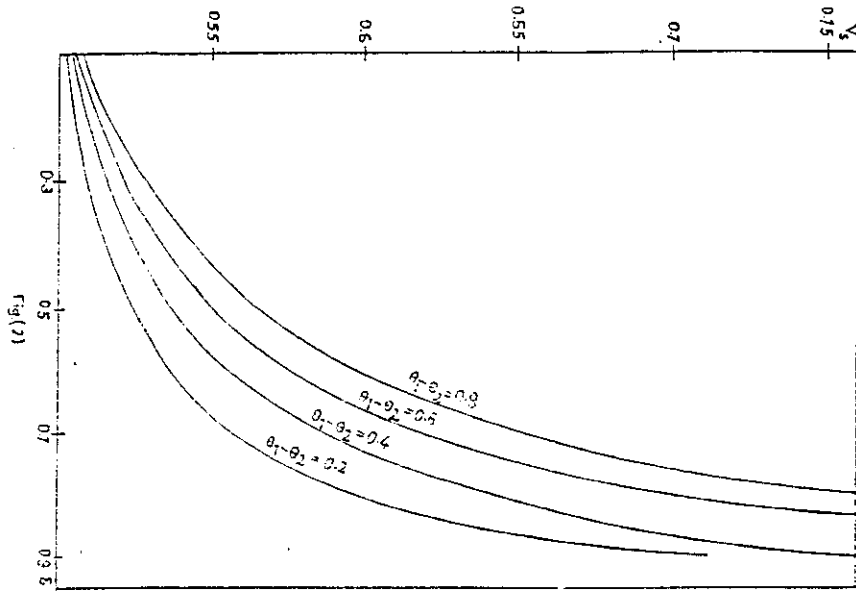
Fig. (9)- The relation between the flow velocity  $V'$  and the distance  $y$  for different values of  $a/b$ ,  $\beta=0.001, \delta=0.5$  and  $\theta_1=\theta_2=0.5$ .

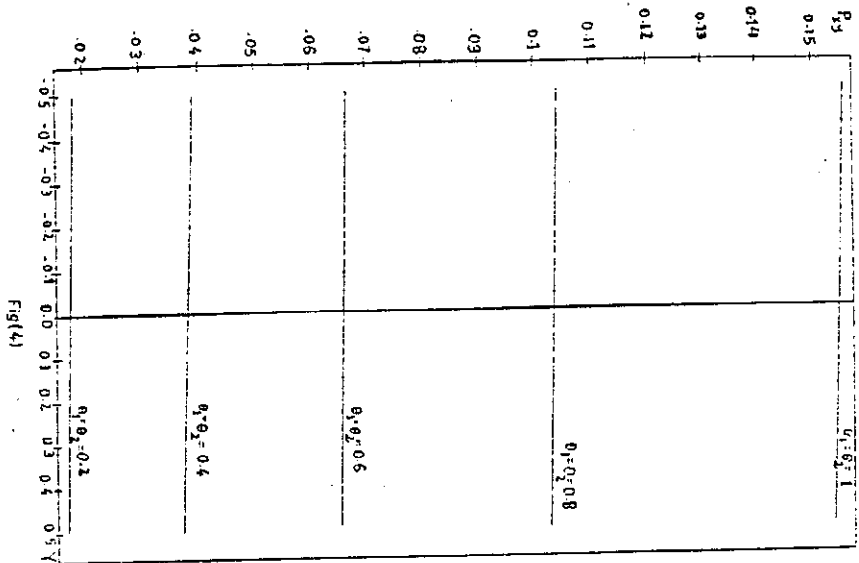
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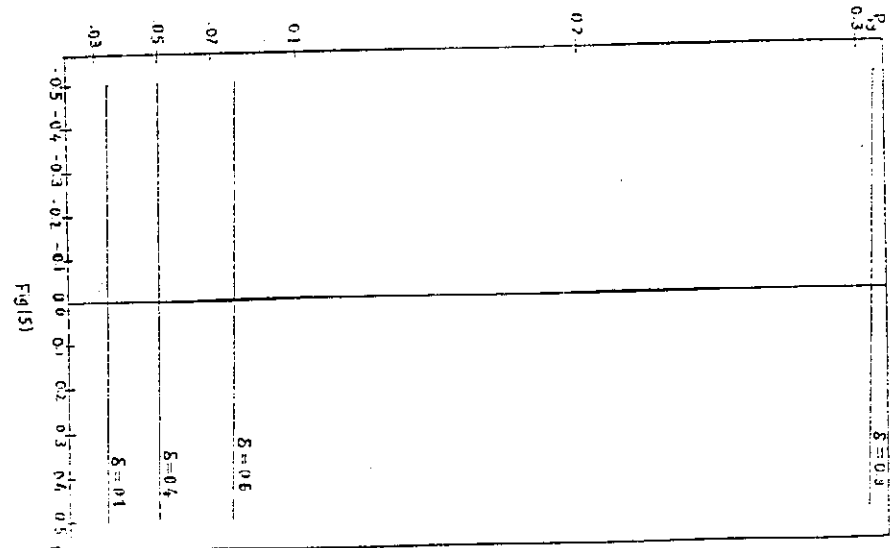




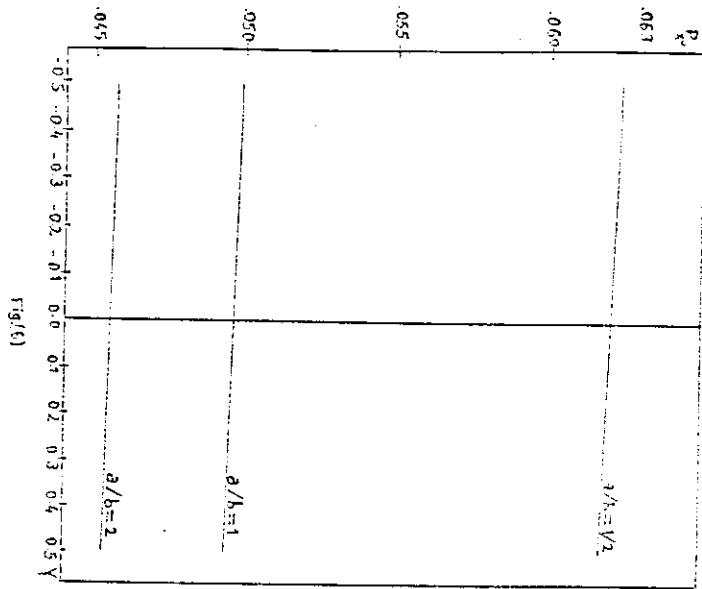




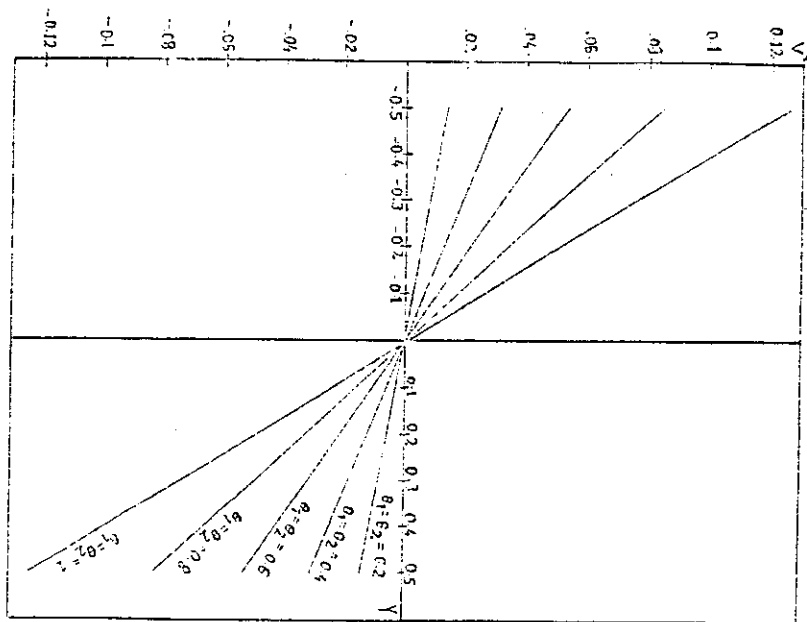
Fig(4)



Fig(5)



Fig(6)



Fig(7)

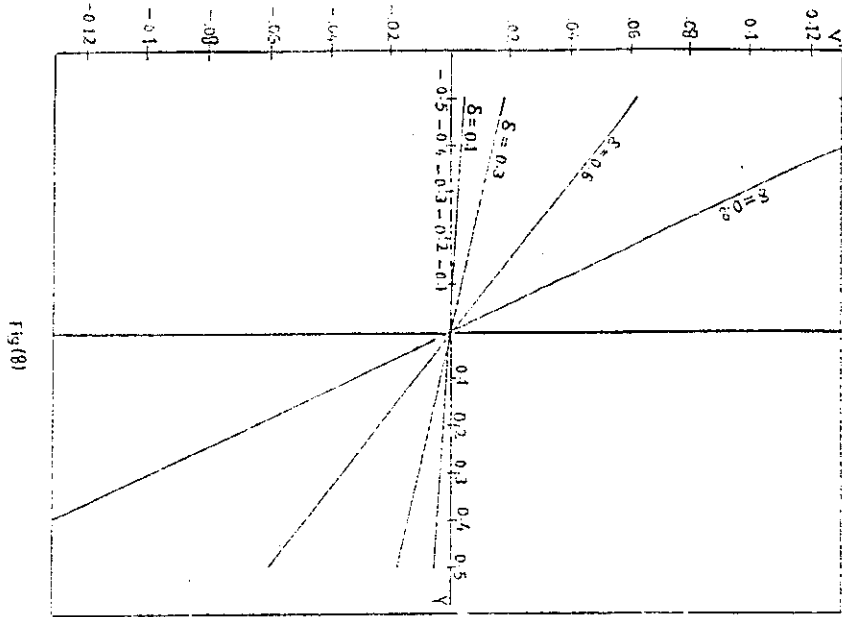


Fig (8)

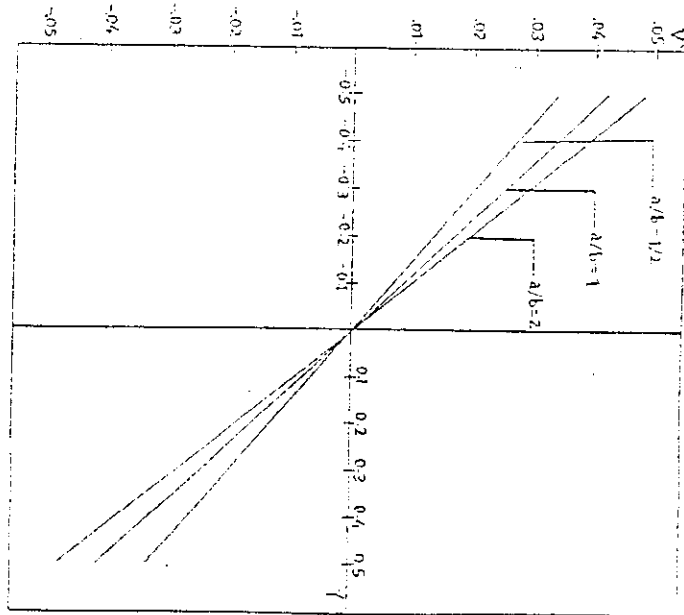


Fig (9)

انسحاب كويبت المنتظم لغاز مخلخل مكون من جسيمات  
مشحونتين لوحين مساميين مع وجود  
معامل انعكاس اختياري

محمود عبد العطي محمود  
كلية العلوم ببناها - جامعة الزقازيق  
قسم الرياضيات

في هذا البحث تم دراسة مسألة انسحاب كويبت المنتظم لغاز مخلخل عند عدد منخفض لمساح في وجود مجال مغناطيسي ثابت ووجود مسامية في اللوحين. مع اعتبار أن معامل الانعكاس في كل من اللوحين العلوي والسفلي مختلفين. وقد تم حل معادلة كرووك للحركة باستخدام طريقة العزوم مع وجود دلتى للتوزيع الجانبية. وقد حصلنا على سرعة انسحاب المائع والسرعة الانزلاكية وقوة اللقص وأيضا معامل الانزلاق كدالة في معامل الانعكاس.

وهنا يجب ملاحظة أننا أهملنا المجال الكهربائي الناشئ من توزيع الجسيمات المشحونة وأيضا المجال الكهربائي الناشئ من تأثير المجال المغناطيسي.

