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ELECTRON BEAM - MODULATED PLASMA INTERACTION

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ABSTRACT

In the present paper we propose a new method of enhancement of a longitudinal wave by injecting an electron beam into a modulated unbounded plasma. The plasma density is modulated by ultrasonic standing wave at wavelength $L = 2\pi / k_0$ such that the ion plasma density can be represented by $N_i = n_{0i} (1 + \beta_i \sin k_0 z)$, where n_{0i} is the ion equilibrium density and β_i is the amplitude of the modulation ($\beta_i \ll 1$). Transverse waves are not affected by plasma modulation while the amplitude of generated longitudinal wave is sharply dependant on β_i and may play an important role for plasma heating.

The injection of electron beam into plasma is of interest in several situations, e.g., wave amplification and generation, plasma heating, plasma instabilities, and particle acceleration (Ivanon, *et al.*, 1984; Gupta, *et al.*, 1988 and Shoucri, *et al.*, 1991). Of these fields, generation of microwaves represents an important application of the collective interaction of beams with plasma (e.g., Wu, *et al.*, 1987). The excitation of waves due to beam-plasma interaction also play an important role to controlling and reducing the attenuation of electromagnetic waves propagating in a plasma-filled waveguide (e.g., Ottinger and Guillory, 1979). Analytical theories on the beam-plasma interaction are preformed without taking the ion response into account, which only valid for weak beam strength. When the beam strength increases or the plasma density is modulated, involving the ions becomes very important.

In this paper, the problem of beam-modulated plasma interaction is investigated. The following assumptions are made for formulating the problem:

1) we apply the non-relativistic hydrodynamics model to both the beam and the plasma,
2) both beam and plasma are cold,
3) ions serving as a neutralizing background with density modulation $n_{0i}\ \beta_i \sin K_0 z$,
4) perturbations are assumed to be of small magnitude so as to make a linear analysis possible,
5) no external magnetic field is applied.

On the basis of these assumptions, the linearized equation of motion, the continuity equation, Maxwell's equations and Gauss law are derived for both the beam and the plasma in the frame of reference attained to the plasma :

$$\frac{\partial \vec{V}_p}{\partial t} = -\frac{e}{m_e}\vec{E},$$

$$\frac{\partial \vec{V}_b}{\partial t} + \vec{V}_{ob}\frac{\partial \vec{V}_b}{\partial z} = -\frac{e}{m_e}[\vec{E} + \frac{1}{c}(\vec{V}_{ob} \times \vec{B})] \qquad (1)$$

$$\frac{\partial n_e}{\partial t} + n_{oe}\frac{\partial V_{ez}}{\partial z} = 0,$$

$$\frac{\partial n_b}{\partial t} + n_{ob}\frac{\partial V_{bz}}{\partial z} + V_{ob}\frac{\partial n_b}{\partial z} = 0 \qquad (2)$$

The subscript p,b refer to the plasma and beam parameters respectively and,

$$N_{pe} = n_{oe} + n_e, \quad N_{pi} = n_{oi}(1 + \beta_i \sin K_o z) \ , \qquad (3)$$

$$N_b = n_{ob} + n_b, \quad \vec{V}_{ep} = 0 + \vec{V}_{ep}, \quad \vec{V}_{ip} = 0, \quad \vec{V}_b = V_{ob}\vec{e}_z + \vec{V}_b$$

The time-independent (or equilibrium) quantities are indicated by zero subscript while the additions represent the perturbed quantities. The previous set of equations give a remarkably accurate description of the small amplitude perturbations ($n_{oe} >> n_e$, $V_{ob} >> V_b$). Putting the perturbed quantities and fields in the form:

$$F = F(z)\, e^{-i\omega t}$$

we obtain

$$n_{ep} = -\frac{i\, n_{oe}}{\omega}\frac{\partial v_{ez}}{\partial z}, \qquad n_b = -\frac{i}{\omega}\left(n_{ob}\frac{\partial v_{bz}}{\partial z} + V_{ob}\frac{\partial n_b}{\partial z}\right) \tag{4}$$

$$\vec{V}_p = -\frac{ie}{m_e\omega}\vec{E}, \qquad \vec{V}_{b(x,y)} = -\frac{ie}{m_e\omega}\left(\frac{E_x \mp \gamma_o B_y}{D_z}\right), \tag{5}$$

$$\vec{V}_{bz} = -\frac{ie}{m_e\omega}\left(\frac{\vec{E}_z}{D_z}\right),$$

where,

$$\gamma_o = \frac{V_{ob}}{c}, \qquad D_z = 1 - i\frac{V_{ob}}{\omega}\frac{\partial}{\partial z}$$

The signs (-,+) in the expression for $V_{b(x,y)}$ indicate X- and Y-components respectively. We also considered the ions to be at rest , $\hat{V}_i = 0$. The current $\vec{J}$ is defined through the plasma and the beam as:

$$\vec{J} = \sum_{\alpha=p,b} e_\alpha N_\alpha \vec{V}_\alpha = -e\,[n_{op}\vec{V}_p + n_{ob}\vec{V}_b + n_b\vec{V}_{ob}] \tag{6}$$

The electric and magnetic fields satisfy Maxwell's equations:

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}, \quad \vec{V} \times \vec{H} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c}\vec{J},$$

We can obtain now the following set of equations inking the fields in their components form:

$$\frac{i\omega}{c}B_x = -\frac{\partial E_y}{\partial z}, \quad \frac{i\omega}{c}B_y = \frac{\partial E_x}{\partial z}, \quad B_z = 0,$$

$$\frac{\partial B_x}{\partial z} = -\frac{i\omega}{c}\epsilon_o E_y - \frac{4\pi e n_{ob}}{c}V_{by},$$

$$\frac{\partial B_y}{\partial z} = \frac{i\omega}{c}\epsilon_o E_x - \frac{4\pi e n_{ob}}{c}V_{bx},$$

$$0 = \frac{i\omega}{c}\epsilon_o E_z - \frac{4\pi e n_{ob}}{c}V_{bz} + \frac{4\pi e V_{ob}}{c}n_b \qquad (7)$$

where,

$$\epsilon_o = 1 - \frac{\omega_p^2}{\omega^2}, \quad \omega_{p,b}^2 = \frac{4\pi e^2 n_{oi,ob}}{m}$$

Using Poisson's equation

$$\vec{\nabla}.\vec{E} = \sum_{\alpha=p,b} e_\alpha N_\alpha,$$

and taking into consideration that the equilibrium charge terms are equal to zero, we get

$$\epsilon_o \frac{\partial E_x}{\partial z} = 4\pi e\, n_{oi}\beta_i \sin k_o z - 4\pi e n_b \qquad (8)$$

Combining this equation with (2) and (5), we derive the following differential equation describes the electric field component E_z

$$\frac{\partial^2 E_z}{\partial z^2} - 2i\chi \frac{\partial E_\chi}{\partial z} - \frac{\chi^2}{\epsilon_o}(\epsilon_o - \frac{\omega_b^2}{\omega^2}) E_z = T(z) \qquad (9)$$

where,

$$\chi = \frac{\omega}{V_{ob}};$$

$$T(z) = \frac{4\pi e n_{oi}\beta_i}{k_o \epsilon_o}[(\chi^2 + k_o^2)\cos k_o z + 2i\chi k_o \sin k_o z]$$

Equation (9) has a solution in the form :

$$E_z = C_1 \exp(i\chi_+ z) + C_2 \exp(i\chi_- z) - \frac{2\pi e n_{oi}\beta_i}{k_o \epsilon_o}[\frac{e^{ik_o z}}{\chi_1^2} - \frac{e^{-ik_o z}}{\chi_2^2}]$$

where,

$$\chi \pm = \chi \pm \frac{\chi_b}{\epsilon_o^{\frac{1}{2}}}, \quad \chi_b = \frac{\omega_b}{V_{ob}};$$

$$(10)$$

$$\chi_{1,2} = [(k_o \mp \chi)^2 - \frac{\chi_b^2}{\epsilon_o}],$$

the -ve and +ve sign are corresponding to X_1 and X_2 respectively. Also by combining (5) and (7) we get the electric field components $E_{x,y}$:

$$\left(\frac{\partial}{\partial z}-i\chi\right)R(z) = 0 , \qquad (11)$$

$$R(z) = \frac{\partial^2 E_{x,y}}{\partial_{z^2}} + K^2 E_{x,y} = 0, \quad K^2 = \frac{\omega^2}{C^2}\left(\epsilon_o - \frac{\omega_b^2}{\omega^2}\right)$$

Solution of (11) may have the form:

$$E_{x,y} = C_{3,5}\exp(iKz) + C_{4,6}\exp(iKz) + \frac{C_o \exp\ (ikz)}{K^2-\chi^2} \qquad (12)$$

It is clear from (10) and (12) that longitudinal waves (E_z) propagating along z-direction, due to the electron beam, has been affected by the plasma modulation. And from (10), we can see that, under the condition $|X_1| > |X_2|$, the longitudinal waves are amplified due to plasma ion modulation, and can be regarded as a source for plasma heating. When taking into consideration the plasma boundaries, modulated plasma will lead to radiation of transverse waves with large amplitude and the plasma may be considered as a source of microwave. This case will be investigated in due course.

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تفاعل حزمة من الإلكترونات مع بلازما معدله

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في هذا البحث أقترح طريقة جديدة لتعزيز الموجة الطولية بواسطة حقن بلازما مع ...ير محدودة بحزمة من الإلكترونات . وقد عدلت كثافة البلازما بواسطة موجة موقوفة ف... ...سمعية طولها الموجي $L = 2\Pi/K$ وعليه يمكن تمثيل كثافة الايونات بالعلاقه

$$N_i = n_{ei} (1 + \beta_i \sin k_o Z$$

حيث n_{ei} كثافة الأيونات المتزنة ؛ β_i سعة التعديل $(1 >> \beta_i)$ وقد و... ...ن الموجات المستعرضة لا تتأثر بالبلازما المعدلة بينما سعة الموجات الطولية تعتمد بحده ... والتي يمكن أن تلعب دورا هاما في تسخين البلازما.