There is an increasing interest in literature to correlate the root mean square radius r_d to other static properties of the deuteron and low energy parameters, e.g., Klarsfeld et al.(1986). Ericson (1984) used the asymptotic S-state wavefunction

$$u \simeq A_s \quad e^{-\gamma r} \qquad (2)$$

where A_s is the asymptotic amplitude and γ^2 is the binding energy in units of fm^{-2} and found that

$$r_d \simeq \frac{1}{4} \gamma^{-3/2} \quad A_s \qquad (3)$$

Ericson (1984) found that an empirical linear relation suggested by (3), holds between r_d and A_s of the potential models. Klarsfeld et al.(1986) considered a larger number of potential models and found that $\partial r_d / \partial A_s = 1.92$.

Another empirical linear relation between r_d and the triplet scattering length a_t of the potential models was also found by Klarsfeld et al. (1986) . The two empirical lines do not pass through the points (r_d, A_s) and (r_d, a_t) representing the experimental values. Almost all of the potential models considered by Klarsfeld et al. (1986) and Ericson (1984) are local. Klarsfeld et al. (1986) concluded that if both of the measurements a_t=5.419 (7) fm

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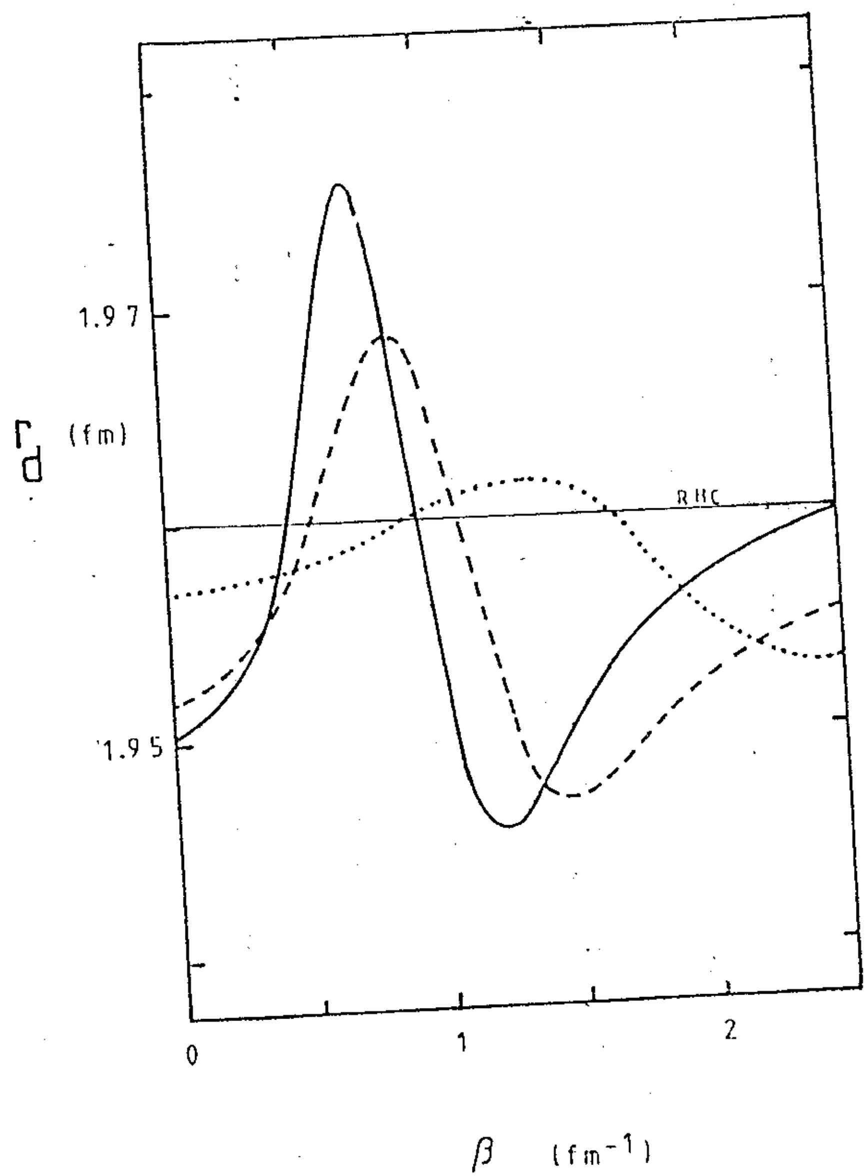

Fig. 3c

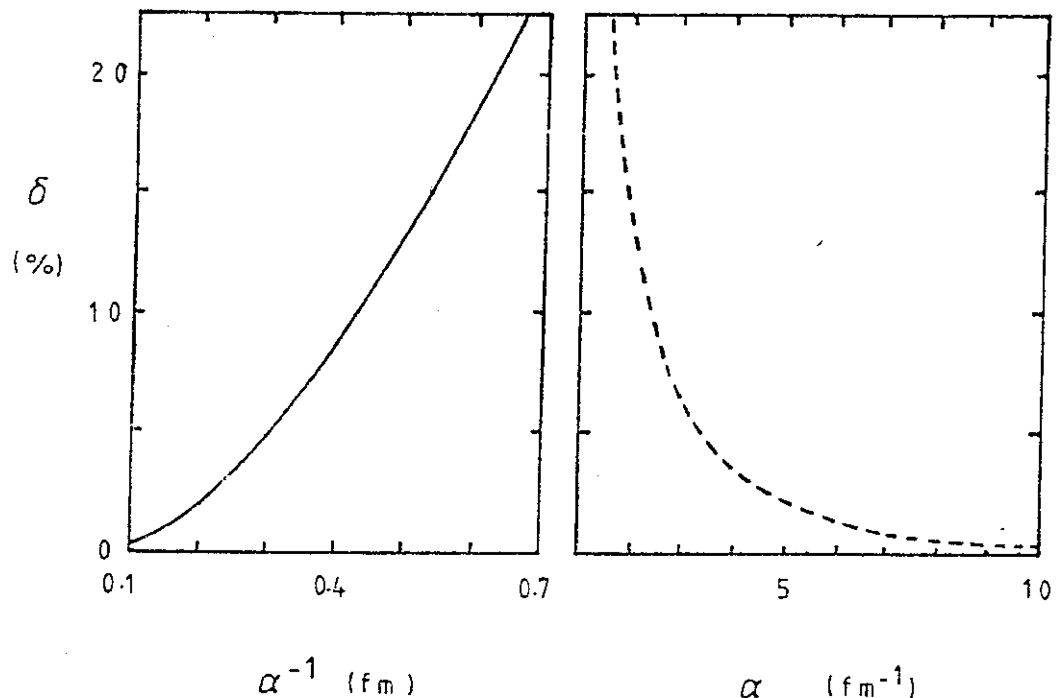

Figure 4

The variation of the maximum percentage change of the root mean square radius γ (see text) versus the "range" α^{-1} (solid line) and α (dashed line).

$$C = [\, 4\alpha^5 / (\alpha^2 - 3\alpha\beta + 3\beta^2)]^{\frac{1}{2}} \qquad (8)$$

The values assumed for α ($\alpha \geqslant 1.5\ fm^{-1}$) insure $\alpha \gg \gamma$ ($\gamma = 0.23\ fm^{-1}$ in equation 2) , and hence, as shown in figure 1,the nonlocal potentials would have the same asymptotic behaviour as the RHC potential Reid(1968).

The variations of r_d of the nonlocal potentials which are phase-equivalent to the Reid hard-core (RHC) potential Reid (1968) versus the nonlocality parameter β ($\alpha = 1.7\ fm^{-1}$), are shown in figure (2). The lack of the dependence of r_d on the nonlocalty which only affects the w-wave is natural since the contribution of the w-wave to r_d is much smaller than that of the u-wave. It is clear from figure (2) that nonlocal potential having the same asymptotic behaviour can have very different values of r_d

Ranges of unitary transformations

The effect found in the previous section is still significant when unitary transformations with relatively smaller ranges (i.e., with larger values of α) are used. The largest changes in the values of r_d become relativley smaller and occur within a wider ranges of β as α is increased as shown in figure 3 .

The values of α between $\alpha = 1.5\ fm^{-1}$ and $\alpha = 9.0\ fm^{-1}$ (corresponding to "ranges" between 0.67 fm and 0.11 fm, respectively) could be used for this purpose. For a given value of α , the root mean square radius r_d is varied against the nomlocality parameter β , and the minimum value r_d^{min} found in this variation is obtained. The influence of the value of the α (and hence , the "range" of the unitary transformation) on obtaining the largest changes in r_d can be seen by calculating δ ; the maximum percentage of the relative change of r_d

$$\delta = \left\{ (r_d^{RHC} - r_d^{min}) / r_d^{RHC} \right\} \quad 100 \qquad (9)$$

where , $r_d^{RHC} = 1.9601$ fm is the root mean square radius of the RHC potential Reid (1968). The variation of δ versus α , in the case of $Z_1 = Z_2 = q$, is shown in figure 4. As α increases , δ will decrease from 22.56 % ($\alpha = 1.5\ fm^{-1}$) to 0.36 % ($\alpha = 9.0\ fm^{-1}$), which is equivalent to 62.28 and 1.00 times the discrepancy between r_d^{RHC} and the experimental value, respectively. This decrease in δ is not unexpected , since the integrand in (1) includes r^2. Hence, nonlocal components with ranges between about 0.11 fm and 0.67 fm could be used .

conclusion

By introducing nonlocalty with short to medium ranges, the values of the root mean squar radius of the deuteron r_d of the local potentials can be varied significantly without changing the asymptotic normalization. Thus, the discrepancies between the points represnting the experimental values of the root mean square radius r_d , the asymptotic S-state amplitute A_s and the triplet scattering length a_t , in one hand and , the empirical linear relations of Ericson (1984) and Klarsfeld et al. (1986), on the other, could be removed.

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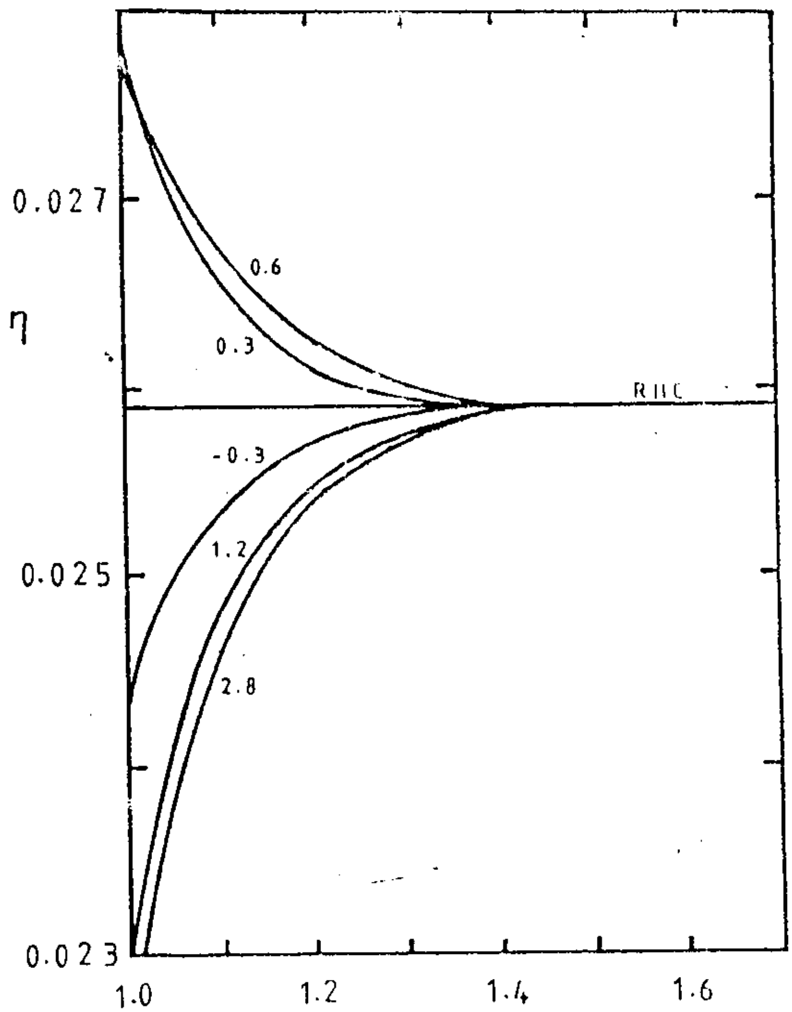

Figure 1

The vaiations of (a) the asymptotic D/S ratio η and (b) the asymptotic S-state amplitude A_s versus α for some values of the nonlocality parameter β . The numbers on the graphs are the values of β . The horizontal lines in (a) and (b) are drawn to show the values of η and A_s of the Ried hard-core potential (Reid 1968) , respectively.

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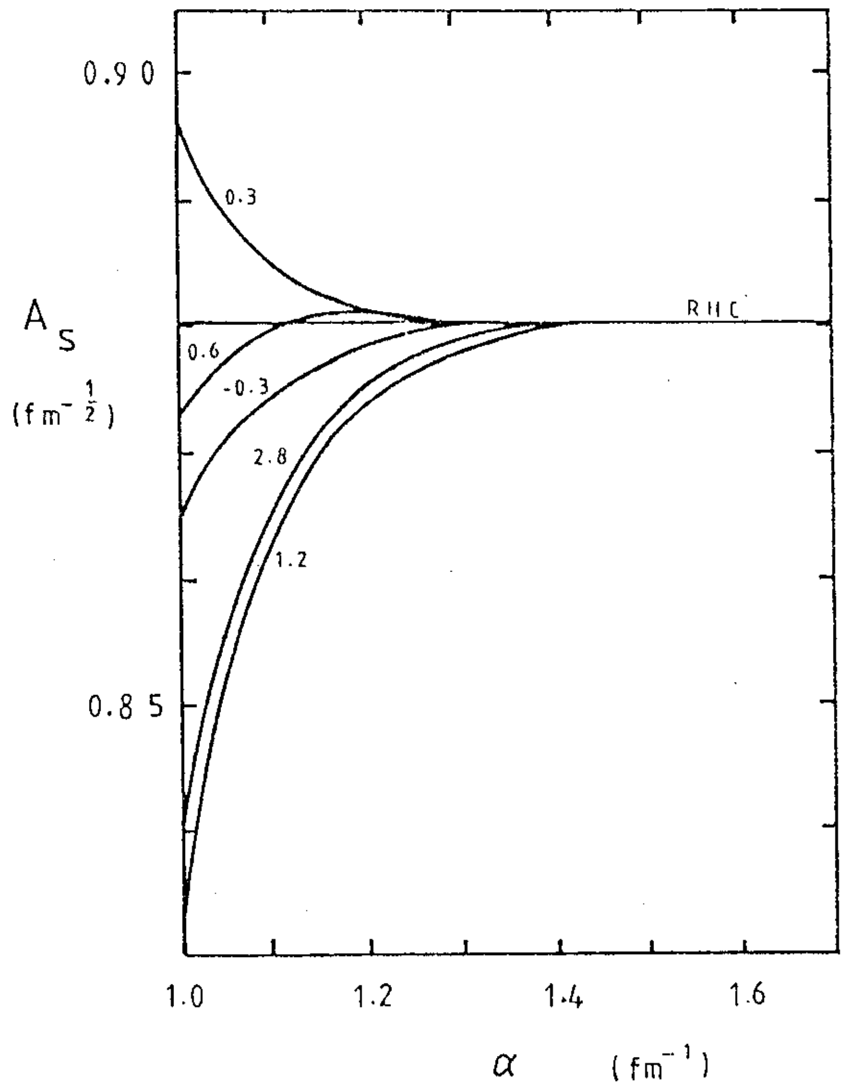

Fig. 1b

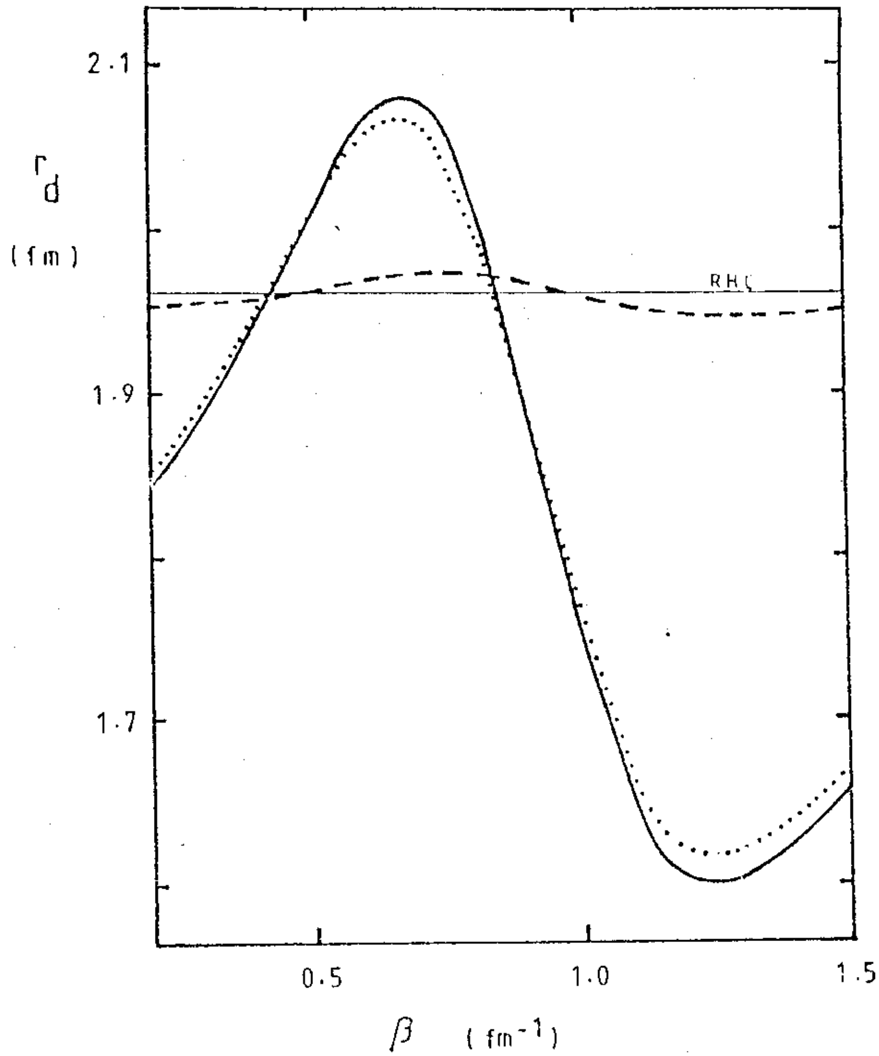

Figure 2

The variation of the root mean square radius of the dueteron r_d versus the nonlocality parameter β , for α =1.7 fm^{-1}, for Z_1 = Z_2 = q (solid line), Z_1= 1 and Z_2 = q (dashed line). The Reid hard-core potenial is the reference potential.

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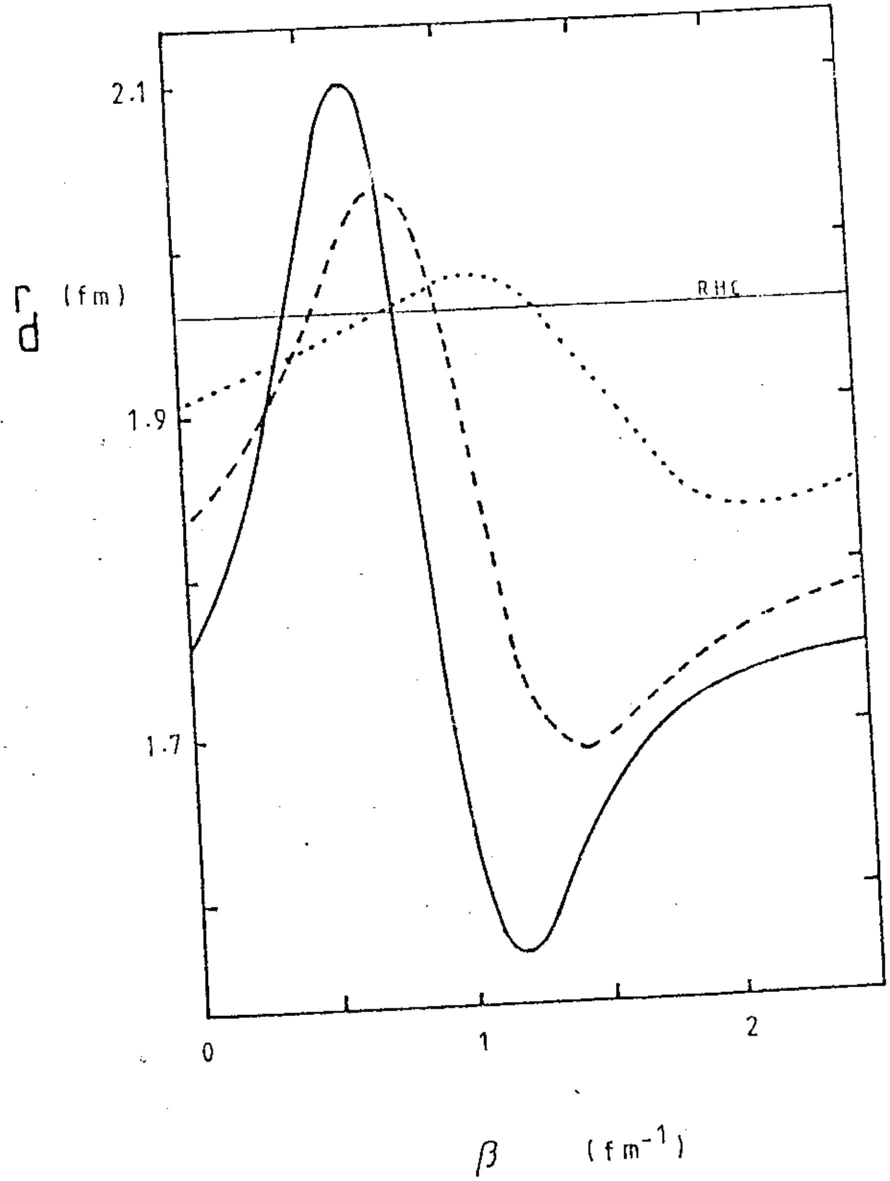

Figure 3

The variation of the root mean squre radius of the deuteron r_d versus β for α =1.6 fm^{-1} (solid lines), α = 2.0 fm^{-1} (dashed lines) and , α = 3.0 fm^{-1}(dotted lines), in the case of (a) $Z_1 = Z_2 = q$, (b) $Z_1 = q$ and Z_2 and Z_2 =1 , and, (c) $Z_1 = 1$ and $Z_2 = q$.

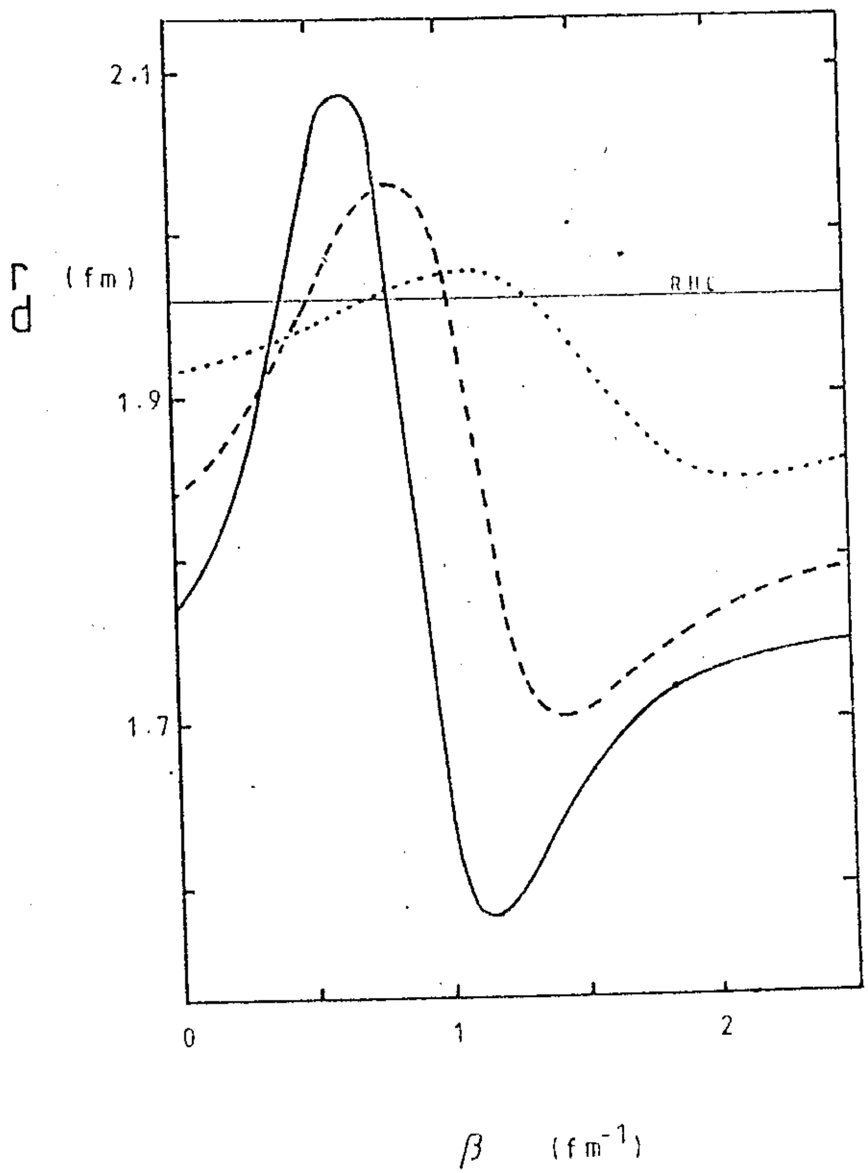

Fig.3b

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Klarsfeld et al. (1984) and r_d = 1.953 (3) fm (Klarsfeld et al. 1986) are correct, then there is something seriously wrong with the phenomenalogical potential models.

It is shown here, that if a potential model incorporates a nonlocal component of appropriate strength and of short to medium range, the value of the root mean square radius of the deuteron r_d could be changed without changing the asymptotic behaviour. This proposes a remedy for this discrepancy . Nonlocal potentials produced by unitary transformations are used for this purpose .

r_d of nonlocal potentials

The root mean square radius r_d is calculated for a class of phase-equivalent nonlocal potentials produced by a short-ranged unitary transformation similar to that used by Kermode et al. (1976) .

$$Z = \begin{bmatrix} Z_u & 0 \\ 0 & Z_w \end{bmatrix} \tag{4}$$

where

$$Z_i = q(\alpha, \beta) = 1 - 2\, g(s)\, g(s') \tag{5}$$

$$g(s) = C\, s\, (1 - \beta s)\, e^{-\alpha s} \tag{6}$$

$i = u$ and w

$s = r - r_c$

Here, r_c is the hard-core radius, C is a normalizing constant such that $\langle g | g \rangle = 1$ and , α and β are the parameters of the nonlocality strength . Three cases are considered,

i) $Z_w = Z_u = g(\alpha, \beta)$; both of the u and w waves are transformed ,

ii) $Z_w = 1$ and $Z_u = q(\alpha, \beta)$; only the u-wave is transformed , and ,

iii) $Z_u = 1$ and $Z_\psi = q(\alpha, \beta)$: only the ψ -wave is transformed .

The transforned deuteron wavefunction is given according to (5) by

$$\psi(r) - 2\,g(r) - 2\,g(r) \int_0^\infty g(s)\,\psi(s)\,ds \qquad (7)$$

where ψ means the u and/or w deuteron radial wavefunctions of the reference local potential (which is taken to be the Reid hard-core (RHC) potential Reid (1968). The explicit from of the normalization constant C of relation (6) is

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فى حـالة نمـاذج الجهـد النـوويـة اللاموضعيـة فقـد وجـد أن جـذر متـوسـط مربع نصف قطـر الديـوترون لا يعتمـد على سـلوك الدوال المـوجيـة عنـد المسـافـات الكبيرة • ووجـد أيضـا أن عـدم التوافـق بين العـلاقـات الخطيـة للجهـود النـوويـة مـع القيـم المقيسـة لا تعنـى أن الجهـود النـوويـة غير صحيحـة •