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EFFECT OF SOURCE SHAPE ON COLLIMATOR SENSITIVITY

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ABSTRACT

Expressions are derived for the penetration factors of the gamma rays through the back corners of the cylindrical collimator when sources of different shapes are used. The sources chosen are point, straight line, circular line and circular plane sources lying in the umbra region of the collimator. The results of the point source are compared to other theoretical calculations. The dependence of the collimator sensitivity on the gamma energy is studied for each of the sources used.

INTRODUCTION

The sensitivity of a collimator is the sum of the sensitivities to the gamma rays which have entered the collimator properly (geometrical case) and those penetrating the corners of the collimator. The penetration factor is then defined as the fractional increase in sensitivity due to penetration.

Calculations of the penetration factors for point gamma ray sources on and off the axis of cylindrical hole collimators have been reported by many authors (Bell and Johnston, 1968 ; Bell et al., 1970 ; Simons et al., 1959 ; Simons, 1962 ; Simons 1964). These calculations are based on the ray tracing technique and the limitations to the expressions obtained have been discussed in details (Bell and Johnston, 1968 ; and Bell et al., 1970). Penetration factors for extended plane gamma sources placed at a distance from a cylindrical collimator have been calculated (El-Kazzaz and Bishara, 1988) by the present author applying the approach of Rotenberg and Johns (Rotenberg and Johns, 1965).

The aim of the present work is to extend the approach of Rotenberg and Johns to calculate the penetration factors in cylindrical collimators when point, straight line, circular line and circular plane gamma sources are used. Then, one can calculate the collimator sensitivity at different gamma energies for each source shape used, Furthermore, the derived formula for the point source is compared to other theoretical formulae.

Calculation of collimator sensitivity.

The sensitivity of a collimator "S" to a source of gamma rays, is related to its geometrical sensitivity "S_o" by the relation (Rotenberg and Johns, 1965) :

$$S = S_o (1 + P) \quad (1)$$

where P is the penetration factor through the corners of the collimator. Values of S_o for the different source shapes used in the present study have been given before Bell and Johnston, 1968 ; and El-Kazzaz et al., 1988). The penetration factor P for each case is calculated applying the approach of Rotenberg and Johns,(1965), based on the idea of shortening the collimator length by a small distance "x" due to penetration of the gamma rays through its corners. This is done as follows:

a- Point source

For a point source of activity N(dps) placed axially at a distance h from the front face of a cylindrical collimator of length t and radius r, the geometrical sensitivity is given by (Bell and Johnston, 1968)

$$S_o = (N/2)\ [1- H(H^2 + r^2)^{-\frac{1}{2}}]$$

where H = t + h. For values of H >> r, the above equation reduces to

$$S_o = Nr^2 / 4H^2 \qquad (2)$$

To calculate the penetration of the gamma rays through the back corners of the collimator consider that the collimator length is shortened by a small distance "X" from the back side. In this case H is replaced by (H - x) and the sensitivity S_x in this case is given by

$$S_x = Nr^2 / 4(H - x)^2$$

The penetration factor P is calculated from the equation Rotenberg and Johns, 1965)

$$P = (1/S_o) \int_{x=0}^{x=2/\mu} (dS_x/dx) . e^{-\mu x} . dx$$

where u is the linear absorption coefficient of the collimator material at the gamma energy used. It can be shown that P in this case is equal to

$$P = [H^2 e^{-2}(H - 2/\mu)^{-2} - 1] + \mu H^2 \int_{x=0}^{x=2/\mu} (H-x)^{-2} e^{-\mu x} dx \qquad (3)$$

It should be mentioned here that the two expressions derived by Simons (Simons 1962) for the present case are:

$$P = 2H^2 e^{-uH} \int_{z=0}^{z=t} (h+z)^{-3} . e^{u(h+z)} . dz$$

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which is referred to as Simons equation A, and used at H>>r. And the approximate formula

$$P = 2! / (\mu H) + 3! / (\mu H)^2 + 4! / (\mu H)^3 + \ldots$$

which is referred to as Simons equation B, and used for $\mu H > 12$.

b- Straight line source

When a straight line source of length L ($\leqslant r$) and activity N(dps) is placed horizontally at a distance h from the collimator, the geometrical sensitivity of the collimator (El-Kazzaz, 1988) is

$$S_o = Nr^2 / 4H.(H^2 + L^2/4)^{\frac{1}{2}} \tag{4}$$

Applying the same method used before to get S_x and P, it can be shown that

$$P = \left(\frac{C_1}{C_2} - 1 \right) + \mu C_1 \int_{x=0}^{x=2/\mu} \frac{e^{-\mu x}.\ dx}{(H-x)\ [(H-x)^2 + L^2/4]^{\frac{1}{2}}} \tag{5}$$

where $C_1 = H(H^2 + L^2/4)^{\frac{1}{2}} .\ e^{-2}$

$C_2 = (H- 2/\mu)\ [(H - 2/\mu)^2 + L^2/4]^{\frac{1}{2}}$

c- Circular line source

In case of a circular line source of radius R ($\leqslant r$) and activity N(dps) lying horizontally so that the axis of the collimator passes through its center, the geometrical sensitivity (El - Kazzaz,1988), is given by

$$S_o = NR^2H / 4(H^2 + R^2)^{3/2} \tag{6}$$

and the penetration factor is then

$$P = \left(\frac{C_3}{C_4} - 1\right) + \frac{\mu}{C_4}\int_{x=0}^{x=2/\mu} \frac{(H-x)\ e^{-\mu x}\ dx}{[(H - x)^2 + R^2]^{3/2}} \qquad (7)$$

where $C_3 = (H - 2/\mu)e^{-2} / [(H - 2/\mu)^2 + R^2]^{3/2}$

$C_4 = H / (H^2 + R^2)^{3/2}$

d- Circular plane source

For a circular plane source of radius R_1 ($<r$) and activity N(dps) lying horizontally at a distance h from the collimator, the geometrical sensitivity is given by (El-Kazzaz, 1988).

$$S_o = (N/2)\ [\ 1 - H(H^2 + R_1^2)^{-\frac{1}{2}}] \qquad (8)$$

and similarly the penetration factor is

$$P = \left(\frac{C_5}{C_6} - 1\right) + \frac{\mu}{C_6}\int_{x=0}^{x=2/\mu} \left[1 - \frac{(H-x)}{((H-x)^2 + R_1^2)^{\frac{1}{2}}}\right] e^{-\mu x}.\ dx \qquad (9)$$

where $C_5 = [1 - (H - 2/\mu)\ ((H - 2/\mu)^2 + R_1^2)^{-\frac{1}{2}}]\ e^{-2}$

$C_6 = [1 - H(H^2 + R_1^2)^{-\frac{1}{2}}]$

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It should be noted that in all the cases studied the gamma source lies in the umbra region of the collimator and therefore the penetration is due to the rays passing only through the back corners of the collimator.

RESULTS AND DISCUSSIONS

Equations 1-9 are used to calculate the sensitivity of the cylindrical collimator to the different source shapes used at the different gamma ray energies. Figure 1 shows a sample of these results for the case where t = h=10 cm. and r = 2 cm. The total activity "N" of each source is taken as one microcurie. It is clear from the figure that the sensitivity increases by about 14-16 % for an increase in energy from 0 to 1.2 MeV. Also it is found that the maximum difference in sensitivity between the four source shapes studied is less than 2 %. This result leads to the conclusion that in such cases one can use the simple formula of the point source to get the sensitivity, at any gamma energy, for other source shapes keeping in mind all the assumptions mentioned before.

Values of the penetration factors calculated from the present formula derived for the point source are compared to those obtained from the two equations of Simons (1962). Figure 2 shows this comparison at the different μH values It is clear that the present results agree to more than 95% with those calculated from Simon equation A.

The problem of penetration of the gamma rays through the corners of the collimator can be looked upon as to cause an apparent increase in the radius of the collimator. Therefore, in equation 2 if S_o is replaced by S and r is replaced by $r+\Delta r$, where Δr is the apparent increase in radius. then it can be shown that:

$$\Delta r/r = (1 + P)^{\frac{1}{2}} - 1$$

Figure 3 shows the dependence of $\Delta r/r$ on the gamma energy at the different values of H.

CONCLUSION

The sensitivity of the cylindrical collimator is calculated, at different gamma energies, for different sources lying in the umbra region of the collimator and was found to be independent of the shape of the source used. The equation derived in the present work for the penetration factors of the point source is compared to other theoretical equations and was found to agree to more than 95% with the exact formula of Simons. The apparent fractional increase in the collimator radius, due to penetration of the gamma rays through the corners, is calculated at different energies and different source to detector distances.

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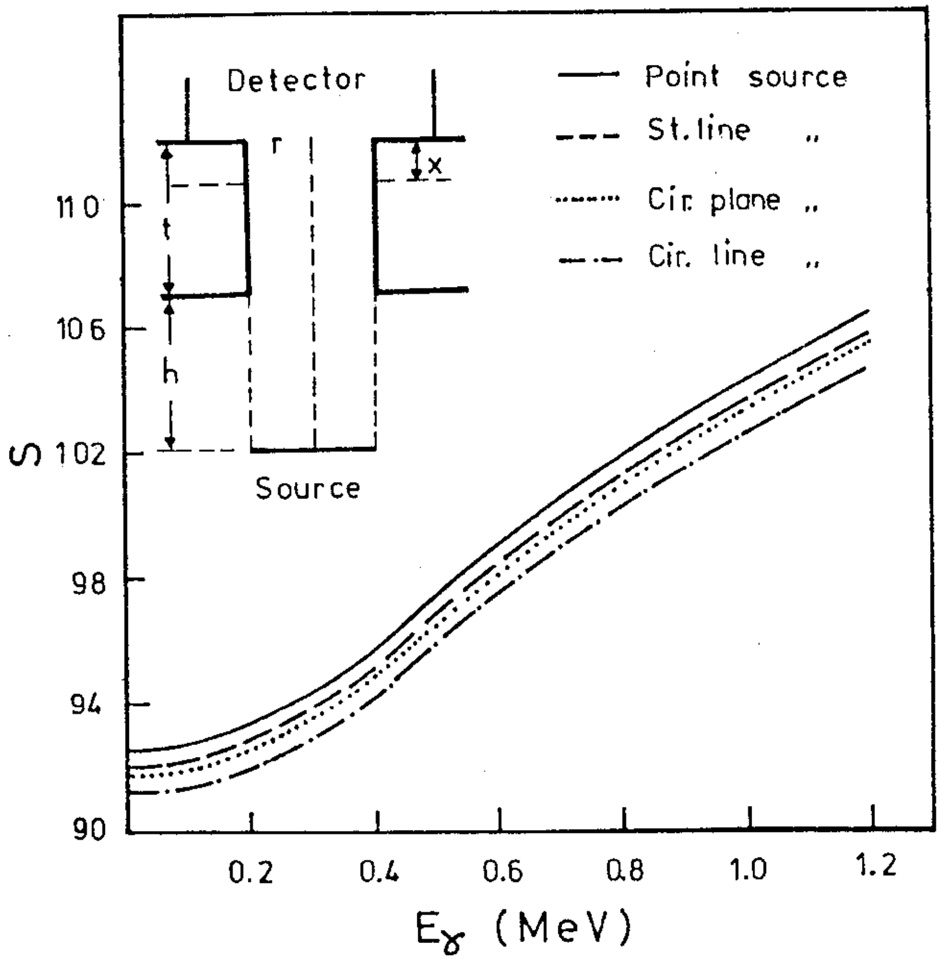

Fig. 1 Sensitivity of the cylindrical collimator at different gamma energies and for different source shapes.

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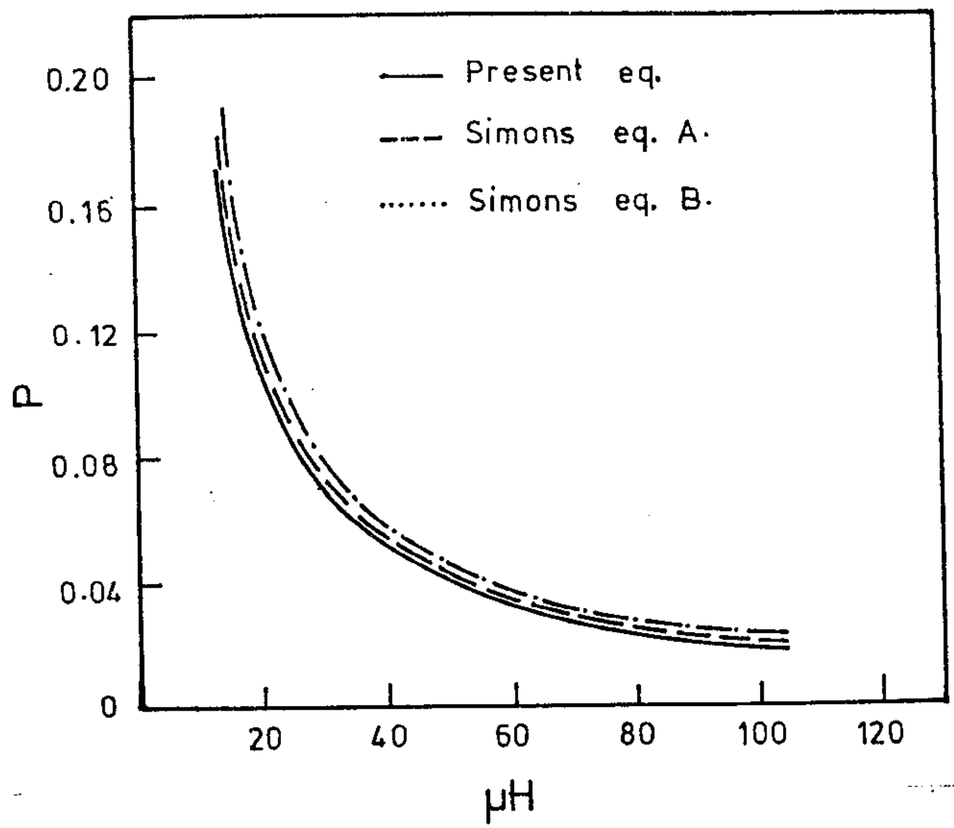

Fig. 2 Comparison of the present calculations of the penetration factors of the point source to other calculations.

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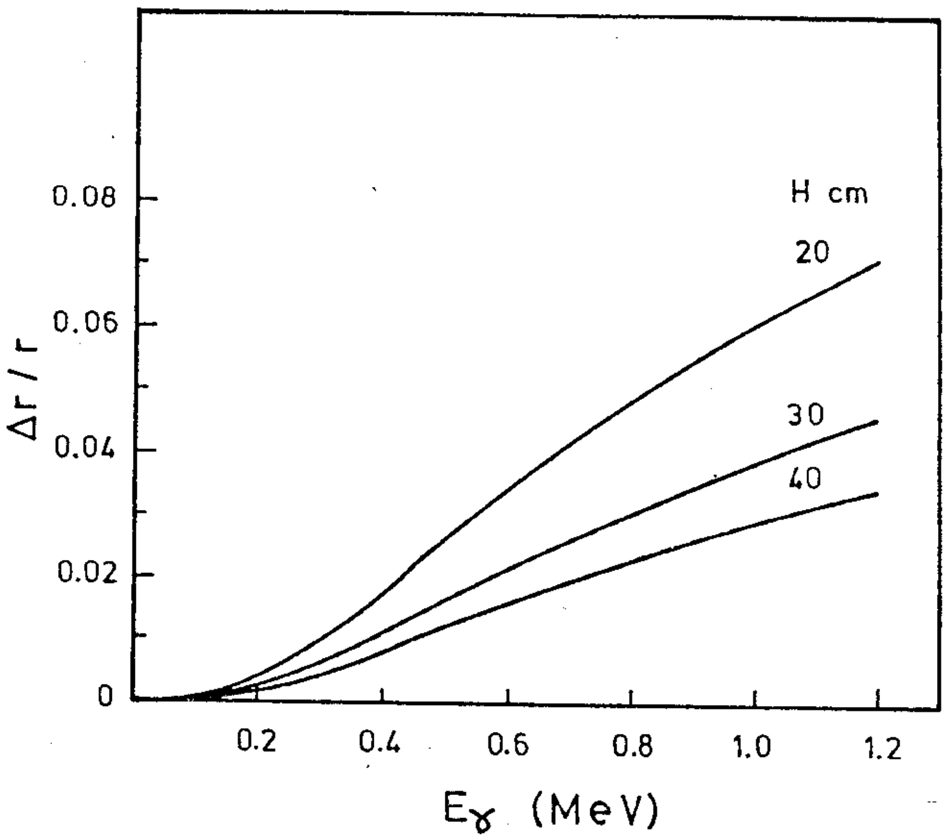

Fig. 3 Variation of the apparent fractional increase in collimator radius with the gamma energy.

تأثير شـكل المصـدر على حساسـية المسدد

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تم فى هذا البحث حسـاب حساسـية المسدد الاسطوانى فى حالة استخدام مصادر مشـعة ذات أشـكال مختلفة وذلك عند الطاقات المختلفـة لأشـعة جاما ووجد أن شـكل المصـدر لا يؤثر على قيمة الحسـاسية عندما يكون المصـدر واقعا فى منطقة الظل الهندسى لفتحة المسدد. كذلك قورنت القيم المسـتنتجة حاليا لمعاملات النفاذ فى حالة المصـدر النقطى بالقيـم المحسوبة من معادلات سابقة. كما تم دراسـة اعتماد الزيادة النسبية الظاهرية فى نصـف قطر المسدد على طاقة أشـعة جاما التى تنفذ خلال جدران المسدد.