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SENSITIVITY ANALYSIS OF MIN-MAX CONTINUOUS STATIC GAMES

BY

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ABSTRACT

We are concerned with sensitivity analysis of min-max continuous static games, aiming to extend the sensitivity analysis results given in [1,2].

Two algorithms, with different techniques are obtained for estimating the first-Karush-Kuhn-Tucker derivatives to a large class of twice differentiable parametric min-max continuous static games. The achievied results are formulated in two lemmas and in two theorems. Furthermore, an illustrative example is given.

INTRODUCTION

Fiacco and Armacost have given in [1,2] the sensitivity analysis results to programs in which the perturbations appear generally. They also have introduced a synthesis of the basic sensitivity theory with classical penalty function theory to estimate such sensitivity information. Two algorithims for estimating the partial parameter derivatives of Karush-Kuhn-Tucker triple have obtained in [1].

In this article we improved such results to be applicable in min-max continuous static games. The obtained results are formulated in two lemmas and two theorems. Furthermore, two algorithms, with different techniques were introduced for estimating the partial parameter derivatives of Karush-Kuhn-Tucker triple. One of the algorithms depends on the sensitivity analysis results and the other, which is prefered from the practical point of view, unifies the basic sensitivity theory with the penalty function theory. Finally, we give an illustrative example.

1. SENSITIVITY ANALYSIS OF A SECOND-ORDER LOCAL SOLUTION

Consider the problem of determination a local solution

$$\hat{u} = (\hat{u}^i \ , \hat{v}) \text{ of}$$

$$\min \ G_i(\hat{x}, u^i, \hat{v}, \varepsilon)$$

$$P_{3M}(\varepsilon) : \text{subject to}$$

$$g(\hat{x}, u^i, \hat{v}, \varepsilon) = 0 \ ;$$

$$h(\hat{x}, u^i, \hat{v}, \varepsilon) \geqq 0$$

and

$$\max \ G_i(\hat{x}, \hat{u}^i, v, \varepsilon)$$

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subject to

$$g(\hat{x},\hat{u}^i,v,\varepsilon) = 0,$$
$$h(\hat{x},\hat{u}^i,v,\varepsilon) > 0.$$

where $x \in E^n$, $u^i \in E^{s_i}$, $v \in E^{s-s_i}$, $(u^i,v) \in E^s$ and $\varepsilon \in E^{\ell}$ (is a parameter vector).

The following theorem presents and demonstrates the relationship of the second-order sufficient optimality conditions to the existence and behaviour of the first -order variations of a local solution and the associated Lagrange multipliers, when the problem functions are subject to parametric variations.

LEMMA 1: (Second order sufficient conditions for a strict local minimizing point of $P_{3M}(o)$ [2]] Assume that the functions defining problem $P_{3M}(o)$ are twice continuously differentiable in a neighborhood of $\hat{u} = (\hat{u}^i,\hat{v})$ then $\hat{u}$ is a strict local minimizing point of problem $P_{3M}(o)$, if there exist (Lagrange multiplier vectors $\hat{\mu}^{(i)} \in E^q$, $\hat{\nu}^{(i)} \in E^n$, $\underline{\hat{\mu}}^{(i)} \in E^q$ and $\underline{\hat{\nu}}^{(i)} \in E^n$ such that the first order Karush-Kuhn-Tucker conditions hold : i.e.

$$h_j(\hat{x},\hat{u}^i,\hat{v},o) \geqslant o \qquad j = 1,2,\ldots,q,$$

$$g_r(\hat{x},\hat{u}^i,\hat{v},o) = o \qquad r = 1,2,\ldots,n,$$

$$\hat{\mu}_j^{(i)} h_j(\hat{x},\hat{u}^i,\hat{v},o) = o, \qquad j = 1,2,\dots,q$$

$$\hat{\underline{\mu}}_j^{(i)} h_j(\hat{x},\hat{u}^i,\hat{v},o) = o$$

$$\hat{\mu}^{(i)} \geqq o$$

$$\hat{\underline{\mu}}^{(i)} \leqq o$$

$$\nabla_x L_i(\hat{x},\hat{u}^i,\hat{v},\hat{\mu}^{(i)},\hat{\nu}^{(i)},0) = \nabla_x G_i(\hat{x},\hat{u}^i,\hat{v},o) - \sum_{j=1}^{q} \hat{\mu}_j^{(i)} \nabla_x h_j(\hat{x},\hat{u}^i,\hat{v},o) + \sum_{=1}^{n} \hat{\nu}_r^{(i)} \nabla_x g_r(\hat{x},\hat{u}^i,\hat{v},o) = o$$

$$\nabla_{u^i} L_i(\hat{x},\hat{u}^i,\hat{v},\hat{\mu}^{(i)},\hat{\nu}^{(i)},o) = \nabla_{u^i} G_i(\hat{x},\hat{u}^i,\hat{v},o) - \sum_{j=1}^{q} \hat{\mu}_j^{(i)} \nabla_{u^i} h_j(\hat{x},\hat{u}^i,\hat{v},o) + \sum_{r=1}^{n} \hat{\mu}_r^{(i)} \nabla_{u^i} g_r(\hat{x},\hat{u}^i,\hat{v},o) = o$$

$$\nabla_x L_j(\hat{x},\hat{u}^i,\hat{v},\hat{\mu}^{(i)},\hat{\nu}^{(i)},o) = \nabla_x G_i(\hat{x},\hat{u}^i,\hat{v},o) - \sum_{j=1}^{q} \hat{\underline{\mu}}_j^{(i)} \nabla_x h_j(\hat{x},\hat{u}^i,\hat{v},o) + \sum_{r=1}^{n} \hat{\underline{\nu}}_r \nabla_x g_r(\hat{x},\hat{u}^i,\hat{v},o) = o$$

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$$\nabla_v L_i(\hat{x},\hat{u}^i,\hat{v},\hat{\underline{\mu}}^{(i)},\hat{\underline{\nu}}^{(i)},o) = \nabla_v G_i(\hat{x},\hat{u}^i,\hat{v},o) - \sum_{j=1}^{q} \hat{\underline{\mu}}_j \nabla_v h_j(\hat{x},\hat{u}^i\hat{v},o) + \sum_{r=1}^{n} \hat{\underline{\nu}}_r \nabla_v g_r(\hat{x},\hat{u}^i,\hat{v},o) = 0$$

and further if for all $Z \neq o$

$Z^T\nabla_x^2 L_i(\hat{x},\hat{u}^i,\hat{v},\hat{\mu}^{(i)}, \hat{\nu}^{(i)},o)Z > o,$

$Z^T\nabla_{u^i}^2 L_i(\hat{x},\hat{u}^i,\hat{v},\hat{\mu}^{(i)},\hat{\nu}^{(i)},o)Z > o;$

$Z^T\nabla_v^2 L_i(\hat{x},\hat{u}^i,\hat{v},\hat{\underline{\mu}}^{(i)}, \hat{\underline{\nu}}^{(i)},o)Z > o;$

$Z^T\nabla_x^2 L_i(\hat{x},\hat{u}^i,\hat{v},\hat{\underline{\mu}}^{(i)}, \hat{\underline{\nu}}^{(i)},o)Z > o.$

such that

$\nabla_{u^i} h_j(\hat{x},\hat{u}^i,\hat{v},o)Z \geqslant o$

$\nabla_v h_j(\hat{x},\hat{u}^i,\hat{v},o)Z \geqslant o$

$\nabla_x h_j(\hat{x},\hat{u}^i,\hat{v},o)Z \geqslant o$ for all j where $h_j(\hat{x},\hat{u}^i,\hat{v},o) = o.$

$\nabla h_j(\hat{x},\hat{u}^i,\hat{v},o)Z = o$ for all j where $\mu_j^{(i)} > o$, $\underline{\mu}_j^{(i)} < o.$

$\nabla_{u^i} h_j(\hat{x},\hat{u}^i,\hat{v},o)Z = o$

$\nabla_v h_j(\hat{x},\hat{u}^i,\hat{v},o)Z = o$

and

$\nabla g_r(\hat{x},\hat{u}^i,\hat{v},o)Z = o \qquad r = 1,2,\ldots,n$

$\nabla_u g_r(\hat{x},\hat{u}^i,\hat{v},o)Z = o$

$\nabla_v g_r(\hat{x},\hat{u}^i,\hat{v},o)Z = o$

Theorem 1 [Basic sensitivity theorem, First order sensitivity results for a second-order local minimizing point $\hat{u} = (u^i,\hat{v})$ of problem $P_{3M}(\varepsilon)$ [1]] If

(i) The functions defining $P_{3M}(\varepsilon)$ are twice continuously differentiable in (x,u) and if their gradients with respect to (x,u) and the constraints are once continuously differentiable in ε in a neighborhood of $(\hat{x},\hat{u},o) = (\hat{x},\hat{u}^i,\hat{v},o)$.

(ii) The second-order sufficient conditions for a local minimum of $P_{3M}(o)$ hold at $(\hat{u})$ with associated Lagrange multipliers $\hat{\mu}^{(i)}$, $\hat{\nu}^{(i)}$, $\hat{\underline{\mu}}^{(i)}$, $\hat{\underline{\nu}}^{(i)}$.

(iii) The gradients $\nabla_{u^i} h_j(\hat{x},\hat{u},o)$ (for j such that $h_j(\hat{x},\hat{u},o) = o$) and $\nabla_{u^i} g_r(\hat{x},\hat{u},o)$ (all r) are linearly independent and

(iv) $\hat{\mu}_j^{(i)} > o$, $\hat{\underline{\mu}}_j^{(i)} < o$ when $h_j(\hat{x},\hat{u},o) = o$.

(j = 1,2,...,q) i.e. strict complementary slackness holds, then

(a) $(\hat{x},\hat{u})$ is a local isolated minimizing point of problem

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$P_{3M}(o)$ and the associated Lagrange multipliers $\hat{\mu}^{(i)}$, $\hat{\nu}^{(i)}$, $\hat{\underline{\mu}}^{(i)}$, $\hat{\underline{\nu}}^{(i)}$ are unique.

(b) for ε in a neighbohood of o, there exists a unique, once continuously differentiable vector function.

$S(\varepsilon) = [x(\varepsilon), u(\varepsilon), \mu^{(i)}(\varepsilon), \underline{\mu}^{(i)}(\varepsilon), \nu^{(i)}(\varepsilon), \hat{\underline{\nu}}^{(i)}(\varepsilon)]$

satisfying the second-order sufficient conditions for a local minimum of problem $P_{3M}(\varepsilon)$ s.t.

$S(o) = ((\hat{x},\hat{u}), \hat{\mu}^{(i)}, \hat{\underline{\mu}}^{(i)}, \hat{\nu}^{(i)}, \hat{\underline{\nu}}^{(i)}) = \hat{S}$ and hence

$(x(\varepsilon), u(\varepsilon))$ is a locally unique local minimum of problem $P_{3M}(\varepsilon)$ with associated unique Lagrange multipliers $\mu^{(i)}(\varepsilon)$, $\underline{\mu}^{(i)}(\varepsilon)$, $\nu^{(i)}(\varepsilon)$, $\underline{\nu}^{(i)}(\varepsilon)$.

(c) for ε near o, the set of binding inequalities is unchanged, strict complementary slachness holds, and the binding constraint gradients are linearly independent at $(x(\varepsilon), u(\varepsilon))$.

2. ESTIMATING THE FIRST KARUSH-KUHN-TUCKER TRIPLE DERIVATIVES TO A LARGE CLASS OF TWICE DIFFERENTIABLE PARAMETRIC MIN-MAX CONTINUOUS STATIC GAMES USING SENSITIVITY ANALYSIS RESULTS.

Let us consider the Karush Kuhn-Tucker first order conditions of $P_{3M}(\varepsilon)$ as follows

$$
\begin{aligned}
&\nabla_x L_i(x,u,\mu^{(i)}, \nu^{(i)}, \varepsilon) = o \\
&\nabla_{u^i} L_i(x,u,\mu^{(i)}, \nu^{(i)}, \varepsilon) = o \\
&\nabla_x L_i(x,u,\underline{\mu}^{(i)}, \underline{\nu}^{(i)}, \varepsilon) = o \\
&\nabla_v L_i(x,u,\underline{\mu}^{(i)}, \underline{\nu}^{(i)}, \varepsilon) = o \\
&\mu_j^{(i)} h_j(x,u,\varepsilon) = o, \quad j = 1,2,\dots,q \\
&\underline{\mu}_j^{(i)} h_j(x,u,\varepsilon) = o \quad j = 1,2,\dots,q \\
&g_r(x,u,\varepsilon) = o \quad r = 1,2,\dots,n
\end{aligned} \tag{1}
$$

From the conditions of Basic sensitivity theorem, system (1) can be differentiated with respect to ε to yield explicit expressions for the first partial derivatives of the vector function $(x,u,\mu^{(i)},\underline{\mu}^{(i)},\nu^{(i)},\underline{\nu}^{(i)}) = (x(\varepsilon), u(\varepsilon), \mu^{(i)}(\varepsilon), \underline{\mu}^{(i)}(\varepsilon), \nu^{(i)}(\varepsilon), \underline{\nu}^{(i)}(\varepsilon))$. as follows:

$$M(\varepsilon) \,.\, \nabla S(\varepsilon) = N(\varepsilon) \tag{2}$$

where M is the Jacobian of (2) w.r.t S, $S = (x,u^i,v,\mu^{(i)}, \nu^{(i)}, \underline{\mu}^{(i)}, \underline{\nu}^{(i)})$ evaluated at $(S(\varepsilon), \varepsilon)$ and N is the negative of the Jacobian matrix of (1) w.r.t. ε calculated at $(S(\varepsilon), \varepsilon)$. Since M is nonsingular for ε near o, it follows that: $\nabla_\varepsilon S(\varepsilon) = M^{-1}(\varepsilon)\ N(\varepsilon)$ (3)

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From this system, the first-order estimate of the variation of an isolated local solution $(x(\varepsilon), u(\varepsilon))$ and the associated unique Lagrange multipliers $\mu^{(i)}(\varepsilon)$, $\nu^{(i)}(\varepsilon)$, $\underline{\mu}^{(i)}(\varepsilon)$, $\underline{\nu}^{(i)}(\varepsilon)$ can be calculated.

3. ESTIMATING THE FIRST KARUSH_KUHN_TUCKER TRIPLE DERIVATIVE TO A LARGE CLASS OF TWICE DIFFERENTIABLE PARAMETRIC MIN_ MAX CONTINUOUS STATIC GAMES USING PENALTY FUNCTIONS.

Altough all parameter derivatives can (in theory) be calculated, as in system (3), and have considerable interest. In practice it is desirable to exploit the fact that it is possible to estimate such sensitivity information from that generated by solution algorithm. So we introduce a synthesis of of the basic sensitivity theory with the classical penalty functions based essentially on approximating a local minimum of problem $P_{3M}(\varepsilon)$ by an unconstrained local minimum $(x(\varepsilon,r), u(\varepsilon,r))$ of $W(x,u,\varepsilon,r)$ and $\underline{W}(x,u,\varepsilon,r)$ for $r>o$ and small that strictly satisfies the inequalities of $P_{3M}(\varepsilon)$.

It is important to present the theorem and lemmas from which we can have the penalty function algorithm as follows:

Theorem 2. [Approximation of first-order sensitivity results and determination of estimates from $W(x,u,\varepsilon,r)$ [1]]

If the assumptions of theorem (1) hold, then in a neighborhood about $(\varepsilon,r) = (o,o)$ there exists a unique once continuously differentiable vector function $[x(\varepsilon,r)$, $u(\varepsilon,r)$, $\mu^{(i)}(\varepsilon,r)$, $\nu^{(i)}(\varepsilon,r)$, $\underline{\mu}^{(i)}(\varepsilon)$, $\underline{\nu}^{(i)}(\varepsilon)]$.

satisfying:

$$\nabla_x L_i(x,u,\mu^{(i)},\nu^{(i)},\varepsilon) = o$$
$$\nabla_x L_i(x,u,\underline{\mu}^{(i)},\underline{\nu}^{(i)},\varepsilon) = o$$
$$\nabla_{u_i} L_i(x,u,\mu^{(i)},\nu^{(i)},\varepsilon) = o$$
$$\nabla_v L_i(x,u,\underline{\mu}^{(i)},\underline{\nu}^{(i)},\varepsilon) = o$$
$$\mu_j^{(i)}\, h_j(x,u,\varepsilon) = r\ ,\ j = 1,2,\ldots,q$$
$$\underline{\mu}_j^{(i)}\, h_j(x,u,\varepsilon) = r\ ,\ j = 1,2,\ldots,q \qquad (4)$$
$$g_j(x,u,\varepsilon) = \nu_j^{(i)}\, r\ ,\ j = 1,2,\ldots,n$$

with $[x(o,o)$, $u(o,o)$, $\mu^{(i)}(o,o)$, $\nu^{(i)}(o,o)$, $\underline{\mu}^{(i)}(o,o)$, $\nu^{(i)}(o,o)] = (\hat{x},\hat{u},\hat{\mu}^{(i)},\hat{\nu}^{(i)},\hat{\underline{\mu}}^{(i)},\hat{\underline{\nu}}^{(i)})$.

and such that for any (ε,r) near (o,o) and $r>o$ $(x(\varepsilon,r)$, $u(\varepsilon,r))$ is a locally unique unconstrained local minimizing point of $W_i(\hat{x},\hat{u}^i,\hat{v},\varepsilon,r)$ and $\underline{W}_i(\hat{x},\hat{u}^i,v,\varepsilon,r)$ with

$$h_j(x(\varepsilon ,r),\ u(\varepsilon,r),\ \varepsilon)) > o \quad j=1,2,\ldots,q$$

and

$$\nabla^2_x\ W_i[x(\varepsilon,r),\ u(\varepsilon,r),\varepsilon,r],\ \nabla^2_{u^i}\ W_i[x(\varepsilon,r),\ u(\varepsilon,r),\varepsilon,\ r]$$

are positive definite

$$\nabla^2_x\ \underline{W}_i[x(\varepsilon,r),\ u(\varepsilon,r),\varepsilon,r),\ \nabla^2_v\ \underline{W}_i[x(\varepsilon,r),\ u(\varepsilon,r),\ \varepsilon,r]$$

are negative definite, where W_i , $\underline{W}_i$ are the Logarithmic-quadratic mixed barrier-penalty functions associated with problem problem $P_{3M}(\varepsilon)$ are given by

$$W_i(\hat{x},u^i,\hat{v},\varepsilon,r) = G_i(\hat{x},u^i,\hat{v},\varepsilon) - r\sum_{j=1}^{q} \ln\ h_j(\hat{x},u^i,\hat{v},\varepsilon) + \frac{1}{2r}\sum_{i=1}^{n} g_i^2\ (\hat{x},u^i,\hat{v},\varepsilon),$$

$$\underline{W}_i(\hat{x},\hat{u}^i,v,\varepsilon,r) = G_i(\hat{x},\hat{u}^i,v,\varepsilon) + \sum_{j=1}^{q} \ln\ h_j(\hat{x},\hat{u}^i,v,\varepsilon) - \frac{1}{2r}\sum g_i^2\ (\hat{x},\hat{u}^i,v,\varepsilon)$$

PROPOSITION 1. [Convergence of penalty function solutions and sensitivity estimates].

If the assumptions of theorem (2) hold, then for $\bar{\varepsilon}$ near o

$$S(\varepsilon,r) \longrightarrow S(\bar{\varepsilon},o) = S(\bar{\varepsilon}),$$

$$\nabla_\varepsilon\ S(\varepsilon,r) \longrightarrow \nabla_\varepsilon\ S(\hat{\varepsilon},o) = \nabla_\varepsilon\ S(\hat{\varepsilon})$$

as $\varepsilon \longrightarrow \hat{\varepsilon}$ and $r \longrightarrow o$.

If we the chain rule, the differentiation of (3) w.r.t. ε yields:

$$M(\varepsilon,r) \quad \nabla_{\varepsilon} S(\varepsilon,r) = N(\varepsilon,r) \tag{5}$$

$$\nabla_{\varepsilon} S(\varepsilon, r) = M^{-1}(\varepsilon,r) . \quad N(\varepsilon,r) \tag{6}$$

An alternative to utilizing (6) to calculate $\nabla_{\varepsilon} S(\varepsilon,r)$ is available, using the fact that the Hessian of $W_i(x,u,\varepsilon,r)$, $\underline{W}_i(x,u,\varepsilon,r)$ are nonsingular at a minimizing point $(x(\varepsilon,r), u(\varepsilon,r)$. This will be seen to have the advantage of involving a smaller matrix inverse rather than the Jacobian matrix of $M(\varepsilon,r)$ appearing in (6) and requires only information that is readily availble from the W_i , $\underline{W}_i$ functions at a minimizing point.

The following algorithm which will be denoned by algorithm I can be used to estimate the first Karush-Kuhn-Tucker triple derivatives to a large class of twice differentiable parametric min-max-continuous static games using penalty functions. This algorithm can be summarized as follows:

ALGORITHM I

1] Minimize the penalty function of problem $P_{3M}(\varepsilon)$ to obtain

$(\hat{x}(\varepsilon,r),\ \hat{u}(\varepsilon,r)$ [in a region such that $h_j(\hat{x}(\varepsilon,r),\ \hat{u}(\varepsilon,r),\varepsilon) > o\ ,\quad j=1,2,\ldots,q$].

2] Substitute by $(\hat{x}(\varepsilon,r),\ \hat{u}(\varepsilon,r))$ in system (4) which is the perturbation of the first order necessary conditions for a local minimizing point of problem $P_{3M}(\varepsilon)$, and solve it to obtain the multipliers $\mu_j^{(i)}(\varepsilon,r),\ \nu_k^{(i)}(\varepsilon,r)$.

3] Calculate $\nabla_\varepsilon[\hat{x}(\varepsilon,r),\ \hat{u}(\varepsilon,r)]$ from the following equations:

$$\nabla_\varepsilon u_i^i(\varepsilon,r) = -\nabla^2_{u^i} W_i[(x(\varepsilon,r),\ u^i(\varepsilon,r),\ \hat{v}(\varepsilon,r),\ \varepsilon,r\ ]^{-1}$$

$$+ \nabla^2_{\varepsilon u^i} W_i[x(\varepsilon,r),\ u^i(\varepsilon,r),\ v(\varepsilon,r),\varepsilon,r]$$

$$\nabla_\varepsilon v(\varepsilon,r) = -\nabla^2_v \underline{W}_i[x(\varepsilon,r),\ u^i(\varepsilon,r),\ v(\varepsilon,r),\ \varepsilon,r]^{-1}$$

$$+ \nabla^2_{\varepsilon v} \underline{W}_i[x(\varepsilon,r),\ u^i(\varepsilon,r),\ v(\varepsilon,r),\varepsilon,r]$$

$$\nabla_\varepsilon x(\varepsilon,r) = -\nabla^2_x W_i[x(\varepsilon,r),\ u^i(\varepsilon,r),\ v(\varepsilon,r),\varepsilon,r]$$

$$+ \nabla^2_{\varepsilon x} W_i[x(\varepsilon,r),\ u(\varepsilon,r),\ v(\varepsilon\ ,r),\varepsilon,r]$$

4] Differentiate the multipliers which were obtained from step [2] w.r.t. ε,

5] Calculate $(\hat{x}(\hat{\varepsilon}),\ \hat{u}(\hat{\varepsilon})),\ \nabla_\varepsilon[(\hat{x}(\hat{\varepsilon}),\hat{u}(\hat{\varepsilon})],\ \hat{\mu}^{(i)}(\hat{\varepsilon}),$ $\hat{\nu}^{(i)}(\hat{\varepsilon}),\ \underline{\hat{\mu}}^{(i)}(\hat{\varepsilon}),\ \underline{\hat{\nu}}^{(i)}(\hat{\varepsilon}),\ \nabla_\varepsilon \hat{u}^{(i)}(\hat{\varepsilon}),\ \nabla_\varepsilon \underline{\hat{\mu}}^{(i)}(\hat{\varepsilon}),\ \nabla_\varepsilon \hat{\mu}^{(i)}(\hat{\varepsilon})$ and $\nabla_\varepsilon \underline{\hat{\nu}}^{(i)}(\hat{\varepsilon})$ by taking the limit of $((\hat{x}(\varepsilon,r),\ \hat{u}(\varepsilon,r)),$

$\nabla_\varepsilon\ [\hat{x}(\varepsilon,r),\ \hat{u}(\varepsilon,r)],\ \hat{\mu}^{(i)}(\varepsilon,r), \hat{\nu}^{(i)}(\varepsilon,r),\ \hat{\underline{\mu}}^{(i)}(\varepsilon,r), \hat{\underline{\nu}}^{(i)}(\varepsilon,r),$
$\nabla_\varepsilon\ \hat{\mu}^{(i)}(\varepsilon,r),\ \nabla_\varepsilon\ \hat{\nu}^{(i)}(\varepsilon,r),\ \nabla_\varepsilon\ \underline{\mu}^{(i)}(\varepsilon,r)$ and $\nabla_\varepsilon\ \hat{\underline{\nu}}^{(i)}(\varepsilon,r)$
when $\varepsilon \to \hat{\varepsilon}$ and $r \to o$.

6] If $[\hat{x}(\bar{\varepsilon}),\ \hat{u}(\bar{\varepsilon}),\ \hat{\mu}^{(i)}(\bar{\varepsilon}),\ \hat{\nu}^{(i)}(\bar{\varepsilon}), \hat{\underline{\mu}}^{(i)}(\bar{\varepsilon}), \hat{\underline{\nu}}^{(i)}(\bar{\varepsilon})]$ is a unique once continuously differentiable vector function, satisfying the second-order sufficient conditions for a local minimum of problem $P_{3M}(\varepsilon)$ stop; otherwise, go to step 1 .

AN ILLUSTRATIVE EXAMPLE.

Consider a two-player game with

$$G_1(.) = (u - 2\ \varepsilon_1)^2 + (r - \varepsilon_2)^2\ ,$$

$$G_2(.) = (u - \varepsilon_1)^2 + \tfrac{1}{2}(v - 2\ \varepsilon_2)^2$$

subject to

$$4\ \varepsilon_1 - u \geqq o\ ,$$

$$4\ \varepsilon_2 - v \geqq o.$$

To determine the first Kuhn-Tucker derivatives to the parametric min-max continuous static games for a player 1. Define the logarithmic-quadratic mixed barrier-penalty function of the player 1 as

$$W = (u - 2\ \varepsilon_1)^2 + (\hat{v} - {}_2)^2 + r[\ln\ (4\ \varepsilon_1 - u) + \ln\ (4\ \varepsilon_2 - v)].$$

$$\underline{W} = (\hat{u} - 2\,\varepsilon_1)^2 + (v - \varepsilon_2)^2 - r[\ln(4\,\varepsilon_1 - u) + \ln(4\,\varepsilon_2 - v)]$$

that uniquely minimized for $r > o$.

$$\nabla_u w = o = 2(u - 2\,\varepsilon_1) - \frac{r}{(4\,\varepsilon_1 - u)} = -2(4 - 2\,\varepsilon_1)(u - 4\varepsilon_1) - r = o$$

$$u(\varepsilon_1, \varepsilon_2, r) = 3\,\varepsilon_1 + (\varepsilon_1^2 - r)^{\frac{1}{2}} \longrightarrow u(\varepsilon_1, \varepsilon_2) = 4\,\varepsilon_1 \text{ as } r \longrightarrow o$$

$$\nabla_v \underline{W} = 2(v - \varepsilon_2) + \frac{r}{(4\,\varepsilon_2 - v)} = 0 = 2(v - \varepsilon_2)(4\,\varepsilon_2 - v) = r$$

$$v(\varepsilon_1, \varepsilon_2, r) = \frac{5}{2}\,\varepsilon_2 + \left(\frac{r}{2} + \frac{9}{4}\,\varepsilon_2\right)^{\frac{1}{2}} \longrightarrow v(\varepsilon_1, \varepsilon_2) = 4\,\varepsilon_2 \text{ as } r \rightarrow$$

Substitute in the perturbed Kuhn-Tucker conditions by

$u(\varepsilon_1, \varepsilon_2, r)$ and $v(\varepsilon_1, \varepsilon_2, r)$

we have

$$\mu_1(\varepsilon_1, \varepsilon_2, r) = \frac{r}{4\,\varepsilon_1 - [3\,\varepsilon_1 + (\varepsilon_1^2 - r)^{\frac{1}{2}}]}$$

$$\mu_2(\varepsilon_1, \varepsilon_2, r) = \frac{r}{4\,\varepsilon_2 - [\frac{5}{2} + (\frac{r}{2} + \frac{9}{4}\,\varepsilon_2)^{\frac{1}{2}}]}$$

It follows that $\mu_1(\varepsilon_1, \varepsilon_2, r) = \mu_1(\varepsilon_1, \varepsilon_2) = -2\,\varepsilon_1$ as $r \longrightarrow o$

and $\mu_2(\varepsilon_1, \varepsilon_2, r)\ \ \mu_2(\varepsilon_1, \varepsilon_2) = o$ as $r \longrightarrow o$

$$\frac{\partial}{\partial \varepsilon_1} u(\varepsilon_1, \varepsilon_2, r) \longrightarrow 4 = \frac{\partial}{\partial \varepsilon_1} u(\varepsilon_1, \varepsilon_2) \quad \text{as } r \longrightarrow o$$

and $\frac{\partial}{\partial \varepsilon_2} v(\varepsilon_1, \varepsilon_2, r) \rightarrow 4 = \frac{\partial}{\partial \varepsilon_2} v(\varepsilon_1, \varepsilon_2)$ as $r \rightarrow 0$

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التحليل الحاسى للمباريات المتصلة الاستاتيكية من نوع min-max

نبيلة أحمد وهيبة

قسم الرياضيات ـ كلية التربية ـ كفر الشيخ

جامعة طنطا ـ مصر

فى هذا البحث نهتم بدراسة التحليل الحاسى للمباريات الاستاتيكية من نوع min-max هادفين الى تعميم نتائج التحليل الحاسى للبرامج غير الخطية (الفياكو وأرماكوست) ، كما يتضمن البحث طريقتين مختلفتين لتقدير المشتقات البارامترية الجزئية الاولى لثلاثى (كوهن ـ تكر) للمباريات المتصلة الوسيطة الاستاتيكية من نوع min-max القابلة للتفاضل مرتين ، علاوة على هذا فقد أعطى مثال توضيحى.