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ON THE GENERATING FUNCTIONS FOR SOME SPECIAL FUNCTIONS AND RELATED TOPICS

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ABSTRACT

In this paper we are concerned with the generating functions for Bessel functions, $J_n(x)$, and Hermite polynomials, $H_n(x)$; accordingly we study two corresponding boundary value problems. The asymptotic forms of $J_n(x)$ and $H_n(x)$ are obtained in order to get the asymptotic formulas for the required generating functions.

Lastly, some applications are given.

INTRODUCTION

Consider a function $F(x , t)$ which gas the following power series expansion

$$F(x , t) = \sum_{n=0}^{\infty} f_n(x) t^n \quad \ldots\ldots\ldots\ldots (1_0)$$

It follows that $F(x , t)$ is a generating function for the set $F_n(x)$. Convergence is not necessary [1] for the relation (1_0) to define the $f_n(x)$.

The generating functions are studied in many works

[1,2]. A large class of these functions for polynomial sets has been investigated in [2].

The considered special functions can be reformulated to represent a solution of the differential equation $-\psi'' + q(z)\psi = s^2\psi$, $z = z(x)$, which has been studied in [3,4,5].

This paper is devoted to find precise asymptotic forms of the considered generating functions $F(x, t)$ for Bessel functions $J_n(x)$ and Hermite polynomils $H_n(x)$. For this purpose we study two boundary value problems for $J_n(x)$ and $H_n(x)$. The corresponding asymptotic formulas for these functions are obtained.

As an application of the obtained results we derive asymptotic forms of the generating function for Laguerre polynomials $L_n^{(\alpha)}(x)$ and get asymptotic formulas for sin x and cos x.

1- An Asymptotic Form of $H_n(x)$.

In this section we obtain an asymptotic formula for $H_n(x)$. Consider the Hermite's differential equation

$$y'' - 2xy' + 2n\, y = 0 \quad , \quad a \le x \le b \qquad \ldots\ldots.(1)$$

subjected to the boundary conditions

$$y(a) = 2^{-\frac{1}{4}}\, e^{\frac{1}{2}a^2} \; , \; y'(a) = 2^{-\frac{1}{4}}\, e^{\frac{1}{2}a^2}\,(i\sqrt{2n} + a), \quad ..(2)$$

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where $n=0,1,2,\ldots$ and a,b are two arbitrary constants.

It is easy to prove the following

Lemma 1. Let $y(x) = 2^{-\frac{1}{4}} e^{\frac{1}{2}x^2} \psi(z)$, $z=\frac{x-a}{T}$ and $T = b - a$.

Then the problem (1) - (2) can be transformed to

$$-\psi'' + q(z)\psi = s^2\psi \; ; \quad 0\leqslant z\leqslant 1, \qquad \ldots\ldots.(3)$$

$$\psi(o\,,\,s) = 1 \quad , \psi'(o\,,\,s) = is, \qquad \ldots\ldots.(4)$$

where $s^2 = 2n\,T^2$ and $q(z) = T^2\,(x^2 - 1)$.

Theorem 1. There exists a +ve number s_o such that the solution of problem (3) - (4) has the asymptotic formula

$$\psi(z\,,\,s) = e^{isz}\,[1 + O(\tfrac{1}{s})\,] \; ; \; |s| > s_o, \qquad \ldots\ldots..(5)$$

or more precisely

$$\psi(z\,,\,s) = e^{isz}\,[1 + \frac{\phi(z)}{is} + O(\frac{1}{s^2})\,] \quad ; \; |s| > s_o, \ldots(6)$$

where $\phi(z) = \frac{1}{2}\int_o^z q(t)dt$ ...(7)

Proof. It can be shown that problem (3) - (4) is equivalent to the integral equation

$$\psi(z\,,\,s) = e^{isz} + \frac{1}{s}\int_o^z \sin s(z - t)\;\; q(t)\;\; \psi(t\,,\,s)dt \qquad \ldots(8)$$

We first prove that if $|s| > s_0$, then $\psi(z , s)$ is bounded in the space $L_2[0 , 1]$. In fact, from (8) we have

$$|\psi| \leqslant 1 + \frac{1}{|s|} \int_0^1 |q(t)| \cdot |\psi(t , s)| \, dt$$

$$\leqslant 1 + \frac{1}{|s|} [\int_0^1 |q(t)|^2 dt]^{\frac{1}{2}} [\int_0^1 |\psi(t , s)|^2 dt]^{\frac{1}{2}}$$

$$= 1 + \frac{1}{|s|} ||q|| \, ||\psi||, \quad ||f|| = [\int_0^1 |f(t)|^2 dt]^{\frac{1}{2}}.$$

This yields

$$||\psi|| < \frac{|s|}{|s| - ||q||} , \quad |s| > ||q|| = s_o.$$

Taking $C = \max_{|s|}\{\frac{|s|}{|s| - ||q||}\}$, it follows that

$$|| \psi(z , s)|| \leqslant C \quad \forall z \in [o , 1] , \quad |s| > s_o \qquad .....(9)$$

Now writing (8) in the form $\psi(z , s)= e^{isz} [1 + f(z , s)]$,

where $f(z , s) = \frac{1}{s} e^{-isz} \int_0^z \sin s(z - t)q(t) \psi(t , s)dt$,

and using (9), we have

$$| f(z , s)| < \frac{C}{|s|} \int_0^1 | q(t) | dt = \frac{C'}{|s|} \text{ (say)}.$$

This proves that $f(z ,s) = O(\frac{1}{s})$ and thus we obtain the required formula (5).

Secondly, let us write (8) in the form

$$\psi(z\,,\,s) = e^{isz}\left[1 + \frac{\phi(z)}{is} + g(z\,,\,s)\right],$$

where $g(z\,,\,s) = -\dfrac{\phi(z)}{is} - \dfrac{e^{-isz}}{2\,is}\displaystyle\int_0^z [e^{is(z-t)} - e^{-is(z-t)}]$

$$\cdot\; q(t)\,\psi(t\,,\,s)dt;$$

here we put $\sin\theta = \frac{1}{2i}[e^{i\theta} - e^{-i\theta}]$. Then using (5), we get $g(z\,,\,s) = -\dfrac{\phi(z)}{is} - \dfrac{e^{-isz}}{2\,is}\displaystyle\int_0^z [e^{is(z-t)} - e^{-is(z-t)}]$

$$\cdot\; q(t)e^{ist}[1 + O(\tfrac{1}{s})]dt$$

$$= -\frac{1}{is}\left[\phi(z) - \frac{1}{2}\int_0^z q(t)dt\right] - I + O\left(\frac{1}{s^2}\right),$$

where $I = \dfrac{1}{2is}\displaystyle\int_0^z q(t)\,e^{-2\,is(z-t)}dt.$

Assume that $q(z)$ has a bounded derivative and integrate I by parts, we find that $I = O(\frac{1}{s^2})$. Hence we obtain

$g(z\,,\,s) = O(\frac{1}{s^2})$, since $\phi(z) = \frac{1}{2}\displaystyle\int_0^z q(t)dt$ from (7).

This completes the proof of the theorem.

Remark. The formula (6) is similar to that obtained by Coddington [3], who used the method of successive approximations of the solution. Here we introduced a new approach of the method of obtaining the formula (6).

Combining the results in lemma (1) and theorem (1), implies the following theorem:

Theorem 2. There exists a + ve integer p such that the Hermite polynomials $H_n(x) = y_n(x)$ have the asymptotic formula $H_n(x) = 2^{-\frac{1}{4}} e^{\frac{1}{2}x^2 + i\sqrt{2n}(x-a)} \left[1 + \frac{x^3-3x-\delta}{6i\sqrt{2n}} + O(\frac{1}{n})\right]$,

$$n \longrightarrow \infty \qquad \ldots\ldots\ldots\ldots(10)$$

where $n > p$ and $\delta = a^3 - 3a$.

Proof. Note that the formula (6) is valid if $|s| > s_o$. This means that $c\sqrt{n} > s_o$, i.e. $n > (s_o^2 / c^2)$. Setting $[s_o^2 / c^2] = p$, where $[x]$ means the greatest integer $\leqslant x$, then the formula (6) is true, if $n > p$.

Scince $\phi(z) = \frac{1}{2} \int_o^z q(z)dz = \frac{1}{2} \int_o^z T^2(x^2 - 1) \frac{dz}{dx} dx$, it is easy to see that

$$\phi(z) = \frac{1}{6} T\ (x^3 - 3x - \delta),\ \delta = a^3 - 3a \qquad \ldots\ldots\ldots\ldots(11)$$

Recalling that $y_n(x) = 2^{-\frac{1}{4}} e^{\frac{1}{2}x^2} \psi(z,n)$, $isz = i\sqrt{2n}(x-a)$ and applying the basic formula (6), we thus obtain the required result (10).

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2- An Asymptotic Form of $J_n(x)$.

In this section we try to find an asymptotic formula for $J_n(x)$. Consider the Bessel's differential equation

$$x^2y'' + xy' + (x^2 - n^2)y = o \quad , a \leqslant x \leqslant b \qquad ....(12)$$

subjected to the boundary conditions

$$y(a) = 1 \qquad , \quad y'(a) = + \frac{n}{a} \quad ; \qquad ....(13)$$

where $n \geqslant o$ and a,b are two arbitrary constants.

It is easy to prove the following

Lemma 2. Let $y(x) = \psi(z)$, $z = \frac{1}{c} \ln \frac{x}{a}$ and $c = \ln \frac{b}{a}$.
Then the problem (12) - (13) is reduced to the following one

$$- \psi'' + q(z)\psi = s^2 \psi \quad , \quad o \leqslant z \leqslant 1 \qquad ....(14)$$

$$\psi(o , s) = 1 \ , \quad \psi'(o , s) = is \qquad ....(15)$$

where $s = - icn$ and $q(z) = -c^2x^2 = -a^2 c^2 e^{2cz}$.

Similarly, we obtain (by theorem (1))

$$\psi(z , s) = e^{isz} [1 + \frac{\phi(z)}{is} + O(\frac{1}{s^2})] \ , \ |s| > s_o, \qquad (*)$$

where $\phi(z) = \frac{1}{2} \int_o^z q(z)dz$ and whence $\phi(z) = \frac{c}{4} (a^2 - x^2)$,

$$x = ae^{cz}.$$

The formula (*) is valid if $|s| > s_o$ i.e. $n > \frac{s_o}{c}$.
Hence this formula is valid if $n > p$, where $p = [s_o/ c]$ is an interger.

Remarking that

$$e^{isz} = e^{cnz} = e^{n\ln\frac{x}{a}} = (\frac{x}{a})^n ,$$

therefore we arrived to the following theorem:

Theorem 3. There exists a + ve integer p such that the Bessel functions $J_n(x)$ have the asymptotic formula

$$J_n(x) = (\frac{x}{a})^n [1 + \frac{a^2 - x^2}{4n} + 0(\frac{1}{n^2})], \qquad n>p \quad ....(16)$$

3- On the Generating Function for $H_n(x)$.

In this section we consider an application of the obtained results in § 1. We apply the asymptotic formula (10) to derive an asymptotic form of the generating function $F(x , t)$ of the form

$$F(x , t) = e^{2tx - t^2} = \sum_{n=o}^{\infty} \frac{H_n(x)}{n!} t^n \quad ....(17)$$

which can be equally written as

$$F(x , t) = \sum_{n=o}^{p} \frac{H_n(x)}{n!} t^n + \sum_{n=p+1}^{\infty} \frac{H_n(x)}{n!} t^n.$$

For the case $n>p$ we apply the asymptotic formula (10), while for $n\leqslant p$ we use the following known formula for $H_n(x)$ [1]:

$$H_n(x) = \sum_{k=o}^{[n/2]} \frac{(-1)^k \; n! \; (2x)^{n-2k}}{k! \; (n-2k)!} = 2^n x^n + P_{n-2}(x);$$
$$n= o,1, ...,p;$$

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in which [m] means the grestest integer < m and $P_{n-2}(x)$ is a polynomial of degree (n - 2) in x.

Then we have

$$F(x\ ,\ t) = \sum_{n=o}^{p} \frac{H_n(x)}{n!} t^n + \sum_{n=p+1}^{\infty} \frac{2^{-\frac{1}{4}}}{n!} e^{\frac{1}{2}x^2 + i\sqrt{2n}(x - a)} \cdot [1 + f(x,n)] t^n ,$$

where $$f(x,n) = \frac{x^3 - 3x - \delta}{6\ i\ \sqrt{2n}} + O(\frac{1}{n}). \qquad \ldots\ldots.(**)$$

$$\alpha_n(x) = 2^n\ x^n + P_{n-2}(x)\ ; \quad n = o,1, \ \ldots\ldots,p \ \ldots.(18)$$

and $$\beta_n(x) = 2^{-\frac{1}{4}}\ e^{\frac{1}{2}x^2 + i\ 2n\ (x-a)} \quad [1 + f(x\ ,\ n)]\ ;\ n= p+1,\ p+2,\ldots \qquad \ldots\ldots\ldots(19)$$

we obtain therefore

$$F(x\ ,\ t) = \sum_{n=o}^{p} \frac{\alpha_n(x)}{n!} t^n + \sum_{n=p+1}^{\infty} \frac{\beta_n(x)}{n!} t^n .$$

Thus, we proved the following theorem:

Theorem 4. The Hermite polynomials possess a generating function F(x , t) which can be written in the following asymptotic form

$$F(x\ ,\ t) = \sum_{n-o}^{\infty} \frac{B_n(x)}{n!} t^n \ , \qquad \ldots\ldots..(20)$$

where $B_n(x)$; n= o,1,2, ..., are given by

$$B_n(x) = \begin{cases} \alpha_n(x) & , \quad 0 \leqslant n \leqslant p \\ \beta_n(x) & , \quad p < n < \infty \end{cases} \qquad \ldots(21)$$

and $\alpha_n(x)$, $\beta_n(x)$ are defined by (18) and (19) respectively.

4- Some Applications on the Asymptotic Form of $H_n(x)$

We apply the obtained results in § 1 to get two asymptotic forms of Laguerre polynomials $L_n^{\alpha}(x)$ with $\alpha = \pm\frac{1}{2}$ and to derive the related generating function.

1- Two Asymptotic Forms of $L_n^{(\alpha)}(x)$ with $\alpha = \pm\frac{1}{2}$

Using the facts [1]

$$\left.\begin{aligned} H_{2n}(x) &= (-1)^n\, 2^{2n}\, n!\; L_n^{(-\frac{1}{2})}(x^2) \\ H_{2n+1}(x) &= (-1)^n\, 2^{2n+1}\, n! \times L_n^{(\frac{1}{2})}(x^2) \end{aligned}\right\} \qquad \ldots\ldots(22)$$

and applying the asymptotic formula (10), we have

$$L_n^{(-\frac{1}{2})}(x) = \frac{(-1)^n}{2^{2n+\frac{1}{2}}\, n!}\; e^{\frac{1}{2}x+2i\sqrt{n}\,(\sqrt{x}-a)}\, \left[1 + \frac{\phi(x)}{12i\sqrt{n}} + O(\tfrac{1}{n})\right], \quad n \longrightarrow \infty \qquad \ldots(23)$$

$$L_n^{(\frac{1}{2})}(x) = \frac{(-1)^n}{2^{2n+\frac{5}{4}}\, n!\, x}\; e^{\frac{1}{2}x + i\sqrt{4n+2}(\sqrt{x}-a)}\left[1+\frac{\phi(x)}{12\, i\sqrt{n}} + O\left(\frac{1}{n}\right)\right],$$

$$n \longrightarrow \infty \quad ..(24)$$

where $\phi(x) = x^3 - 3x - \delta$. The last two formulas are true if $n>p$ and p is a +ve integer.

2- On the generating functions for $L_n^{(\alpha)}(x)$, $\alpha = \pm\frac{1}{2}$

The Laguerre polynomials $L_n^{(\alpha)}(x)$ are defined by means of a generating function of the form

$$F^{(\alpha)}(x,t) = \Gamma(1+\alpha)\,(xt)^{-\frac{\alpha}{2}}\, e^t\, J_\alpha(2\sqrt{xt}) = \sum_{n=o}^{\infty} \frac{L_n^{(\alpha)}(x)}{(1+\alpha)_n}\, t^n ,$$

$$............(25)$$

where $(\alpha)_n = \alpha(\alpha+1)(\alpha+2) \;.....(\alpha+n-1), n\geqslant 1;\; (\alpha)_o = 1$ if $\alpha \neq o$, $J_\alpha(u)$ is a Bessel function of index α and $\Gamma(u)$ is the gamma function.

Now (15) may be written in the form

$$F^{(\alpha)}(x,t) = \sum_{n=o}^{p} \frac{L_n^{(\alpha)}(x)}{(1+\alpha)_n}\, t^n + \sum_{n=p+1}^{\infty} \frac{L_n^{(\alpha)}(x)}{(1+\alpha)_n}\, t^n ,$$

$$\alpha = \pm\tfrac{1}{2}.$$

For the case $n>p$ we apply the asymptotic formulas (23) and (24), while for $n\leqslant p$ we use the following formula

[1] for $L_n^{(\alpha)}(x)$:

$$L_n^{(\alpha)}(x) = \sum_{k=o}^{n} \frac{(-1)^k \, (1+\alpha)_n \, x^k}{k!(n-k)! \, (1+\alpha)_k} = \frac{(-1)^n}{n!} x^n + P_{n-1}^{(\alpha)}(x) ,$$

$n= o,1, \ldots,p$

where $P_{n-1}^{(\alpha)}(x)$ is a polynomial of degree $(n-1)$ in x.

Putting

$$\gamma_n^{(\alpha)}(x) = \frac{(-1)^n}{n!} x^n + P_{n-1}^{(\alpha)}(x) \; ; \quad n = 0,1, \ldots,p, \tag{26}$$

$$\delta_n^{(\alpha)}(x) = h_n^{(\alpha)}(x) \left[1 + \frac{\phi(x)}{12 \, i\sqrt{n}} + O(\frac{1}{n})\right]; \quad n=p+1, p+2, \ldots \tag{27}$$

and

$$h_n^{(\alpha)}(x) = \begin{cases} \dfrac{(-1)^n}{2^{2n+\frac{1}{4}} \, n!} \, e^{\frac{1}{2}x + 2i\sqrt{n}(\sqrt{x}-a)} & , \text{ if } \alpha = -\frac{1}{2} \\[2ex] \dfrac{(-1)^n}{2^{2n+\frac{5}{4}} \, n!} \, e^{\frac{1}{2}x + i\sqrt{4n+2}(\sqrt{x}-a)} . \dfrac{1}{x} & , \text{ if } \alpha = +\frac{1}{2} \end{cases}$$

we obtain therefore

$$F^{(\alpha)}(x,t) = \sum_{n=o}^{p} \frac{\gamma_n^{(\alpha)}(x)}{(1+\alpha)_n} t^n + \sum_{n=p+1}^{\infty} \frac{\delta_n^{(\alpha)}(x)}{(1+\alpha)_n} t^n .$$

Thus, we proved the following theorem :

Theorem 5- The Laguerre polynomials $L_n^{(\alpha)}(x)$, $\alpha = \pm\frac{1}{2}$ have a genersting function $F^{(\alpha)}(x,t)$, which can be expressed asymptotically in the form

$$F^{(\alpha)}(x,t) = \sum_{n=o}^{\infty} \frac{C_n^{(\alpha)}(x)}{(1+\alpha)_n} t^n , \quad \alpha = \pm\tfrac{1}{2}, \qquad \text{..........(28)}$$

where $C_n^{(\alpha)}(x)$; n=o,1,2,.......are given by

$$C_n^{(\alpha)}(x) = \begin{cases} \gamma_n^{(\alpha)}(x) , & 0 \leqslant n \leqslant p \\ \delta_n^{(\alpha)}(x) , & p < n < \infty \end{cases} \qquad \text{..........(29)}$$

and $\gamma_n^{(\alpha)}(x)$, $\delta_n^{(\alpha)}(x)$ are defined by (26) and (27) respectively.

5- On the generating function for $J_n(x)$.

In this section we give an application of the obtained results in §2. We apply the asymptotic formula (16) to derive an asymptotic form of the generating function F(x,t) for $J_n(x)$.

Bessel functions $J_n(x)$ have the generating function

$$F(x,t) = e^{\frac{1}{2}x(t-t^{-1})} = \sum_{n=-\infty}^{\infty} J_n(x)t^n , \qquad \text{..........(30)}$$

which by virtue of the fact $J_{-n}(x) = (-1)^n J_n(x)$ reads

$$F(x,t) = J_o(x) + \sum_{n=1}^{\infty} J_n(x) [t^n + (-1)^n t^{-n}]$$

$$= J_0(x) + \sum_{n=1}^{p} J_n(x)[t^n + (-1)^n t^{-n}] + \sum_{n=p+1}^{\infty} J_n(x) \cdot [t^n + (-1)^n t^{-n}].$$

For the case $n>p$ we apply the asymptotic formula (16), while for $n\leq p$ we use the following famous formula for $J_n(x)$ [1] :

$$J_n(x) = \sum_{k=o}^{\infty} \frac{(-1)^k (\frac{1}{2}x)^{n+2k}}{k!\ (n+k)!}, \quad n \text{ integer} \quad ......(31)$$

Then we have

$$F(x,t) = J_0(x) + \sum_{n=1}^{p} J_n(x)\ [t^n + (-\frac{1}{t})^n] +$$
$$\sum_{n=p+1}^{\infty} (\frac{x}{a})^n\ [1 + \frac{a^2 - x^2}{4\ n} + 0(\frac{1}{n^2})]\ [t^n + (\frac{-1}{t})^n].$$

If we put now

$$a_n(x) = \sum_{k=0}^{\infty} \frac{(-\frac{1}{2}x)^k}{k!\ (n+k)!}, \quad n=1,2,\ ....,p \qquad (32)$$

and

$$b_n(x) = a^{-n}\ [1 + \frac{a^2 - x^2}{4n} + 0(\frac{1}{n^2})]\ ; \quad n=p+1\ ,\ p+2, \quad ...(33)$$

we obtain

$$F(x,t) = J_o(x) + \sum_{n=1}^{p} a_n(x)\ [(xt)^n + (-\frac{x}{t})^n] + \sum_{n=p+1}^{\infty} b_n(x)\ [(xt)^n + (-\frac{x}{t})^n]$$
$$...............(34)$$

Consequently, we proved the following theorem :

Theorem 6. The generating function for the Bessel functions has the following asymptotic form

$$F(x,t) = J_o(x) + \sum_{n=1}^{\infty} A_n(x) \left[(xt)^n + \left(-\frac{x}{t}\right)^n\right], \qquad \text{..........(35)}$$

where $A_n(x)$; $n= 1,2, \ldots\ldots$ are given by

$$A_n(x) = \begin{cases} a_n(x) , & 1 \leqslant n \leqslant p \\ b_n(x) , & p < n < \infty \end{cases} \qquad \text{..........(36)}$$

and $a_n(x)$, $b_n(x)$ are defined by (32) and (33) respectively.

6- Asymptotic Forms of sin x and cos x.

In this section we apply the results obtained in § 4. That is, using the formula (35) for the generating function we can derive the asymptotic forms of sin x and cos x.

Now, by definition, the generating function for Bessel functions is $F(x,t) = e^{\frac{1}{2}x(t-t^{-1})}$ and hence putting $t_x = i$ we then conclude that

$$F(x,i) = e^{ix} = \cos x + i \sin x \qquad \text{.........(37)}$$

On the other hand, using (35) with t=i we get

$$F(x,i) = J_o(x) + \sum_{n=1}^{\infty} A_n(x) \left[(ix)^n + \left(-\frac{x}{i}\right)^n\right]$$

$$= J_o(x) + 2 \sum_{n=1}^{\infty} A_n(x) \;.\; (ix)^n \qquad \text{.........(38)}$$

and this relation can be written in the form

$$F(x,i) = J_o\,(x) + 2 \sum_{k=1}^{\infty} A_{2k}\,(x).\,(ix)^{2k} + 2 \sum_{k=o}^{\infty} A_{2k+1}\,(x).\,(ix)^{2k+1},$$

i.e.

$$F(x,i) = J_o\,(x) + 2 \sum_{k=1}^{\infty} (-1)^k A_{2k}(x)\, x^{2k} + 2\,i \sum_{k=o}^{\infty} (-1)^k A_{2k+1}(x) x^{2k+1} \qquad \text{..............(39)}$$

Therefore, from (37) and (39) we finally obtain the following theorem

Theorem 7. $\cos x = J_o(x) + 2 \sum_{k=1}^{\infty} (-1)^k A_{2k}(x)\, x^{2k}$ ....(40)

and

$$\sin x = 2 \sum_{k=o}^{\infty} (-1)^k A_{2k+1}(x) x^{2k+1}, \qquad \text{........(40')}$$

where $A_n(x)$; n=1,2,3, ... are given by (36).

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دراسة الدوال المولدة لبعض الدوال الخاصة

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لقد تم فى هذا البحث دراسة الدوال المولدة لدوال بسل ودوال هيرميت. ولهذا الغرض قد قام الباحث بدراسة مسألتين خاصتين حديثتين لتلك الدوال وفى نفس الوقت قد تم الحصول على صيغ تقاربيه للدوال المذكورة.

وكتطبيق على النتائج السابقه ، توصل البحث الى ايجاد صيغ تقاربيه لدوال لاجير وكذلك لدوالها المولدة.