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A NOTE ON: " THE DEGREE OF ISOMETRIC FOLDINGS OF RIEMANNIAN MANIFOLDS WITH BOUNDARY "

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A map $f : M \rightarrow N$, where M and N are smooth Riemannian manifolds of dimensions m and n respectively, is said to be an isometric folding of M into N iff for any piecewise geodesic path $\gamma: J \rightarrow M$, the induced path $f \circ \gamma : J \rightarrow N$ is a piecewise geodesic and of the same length. The set of all points $x \in M$ at which f is not differentiable is called the set of all singularities of f and is denoted by $\sum f$. This set decomposes M into mutually disjoint connected totally geodesic submanifolds which are called strata [1].

Consider an isometric folding $f : M \rightarrow N$ for which the dimensions of M and N are equal. Suppose that both M and N are oriented, and that M is compact without boundary. Thus, Hopf degree of f (deg f) is well defined and it is either ± 1 or 0, [2].

Now, let M be a smooth Riemannian manifold with boundary $\partial M = Q$, and suppose that $f : M \longrightarrow M$ is an isometric folding. Let $\mathcal{F}(M)$ denotes the set of all isometric foldings of M into itself, and $\mathcal{F}(M,Q)$ the set of all isometric foldings of M into itself that preserve the boundary $\partial M = Q$. Then there is an inclusion $\mathcal{F}(M,Q) \longrightarrow \mathcal{F}(M)$, and a homomorphism

$$\mathcal{F}(M) \supset \mathcal{F}(M,Q) \xrightarrow{\theta} \mathcal{F}(Q) \text{ given by } \theta(f) = f/Q \cdot$$

Suppose that M is compact, connected and oriented, and that $f \in \mathcal{F}(M)$ is such that $f/Q \in \mathcal{F}(Q)$. Then Q has an induced orientation.

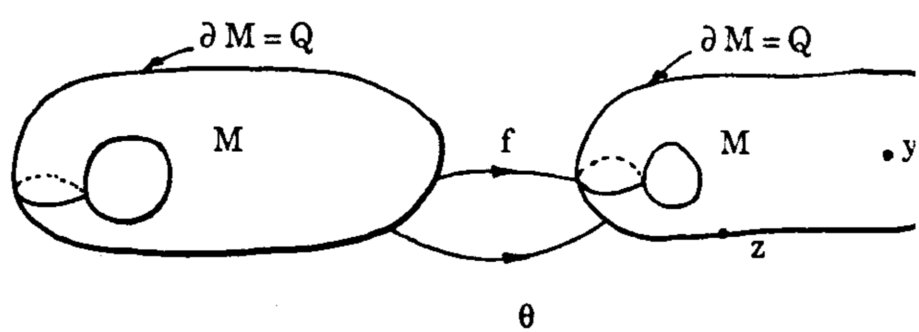
In this paper we show that both $f \in \mathcal{F}(M,Q)$ and f/Q have the same degree.

Main result:

Let M be a smooth oriented compact Riemannian manifold with an oriented compact boundary Q.

Theorem: If $f \in \mathcal{F}(M,Q)$ has degree $k, k = 1, -1, 0$. Then $\theta(f) = f/Q \in \mathcal{F}(Q)$ has also degree k.

Proof:



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Let Y be any regular value for f, and suppose that $f^{-1}(y)$ contains λ points of positive type and μ points of negative type . So that [2] $\deg(f;y) = \lambda - \mu = k$.

From the inverse function theorem [3], we can find an open set $U \subset M$ about y and an open set V (x) about each x $f^{-1}(y)$ such that f maps each V(x) diffeomorphically onto U preserving or reversing orientation according to the type of x.

Now let z be a point in the boundary of M, i.e.,Q. If z is a regular value of f, then $f^{-1}(z)$ will contain λ' points of positive type and μ' points of negative type such that $\lambda' - \mu' = \deg f = k$. Since f preserves boundary of M, thus $\lambda' - \mu' = \deg f = \deg (f/Q) = \deg \theta (f)$.

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درجة دالة الطى الأيزومترى لمتعددات الطى الريمانية ذات المحيط

الدكتوره ، انتصار الخولى

فى هذا البحث أثبت أن درجة دالة الطى الأيزومترى لمتعدد طيات ريمانى له محيط تكون مساوية لدرجة دالة الطى الأيزومترى لمحيط متعدد طيات ريمانى شريطة أن تكون الدالة الأولى ترسل المحيط لنفسه.